Bath induced phase transition in a Luttinger liquid

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Motivation : Localisation and dissipation

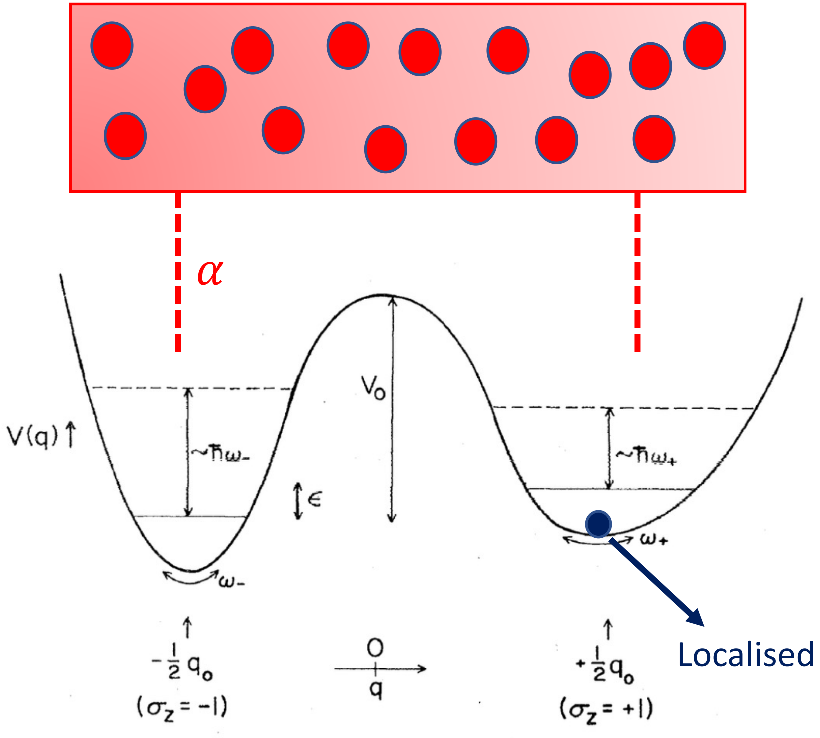
- Bray and Moore (PRL,1982),Caldeira and Leggett (Physica A, 1983) : Dissipative two-state systems.

  System : single spin-1/2 particle in constant magnetic field along x and z direction

  Bath : collection of simple harmonic oscillators

Spectral function of the bath :

$$J(\Omega) \approx \pi\alpha\Omega^{s} \quad \text{(for small } \Omega\text{)}$$



- s = 1 : ohmic bath
- 0< s < 1 : subohmic bath $\Longrightarrow$ Localisation of system!

Question : What happens in one dimension?

Description of the microscopic system

Hamiltonian:

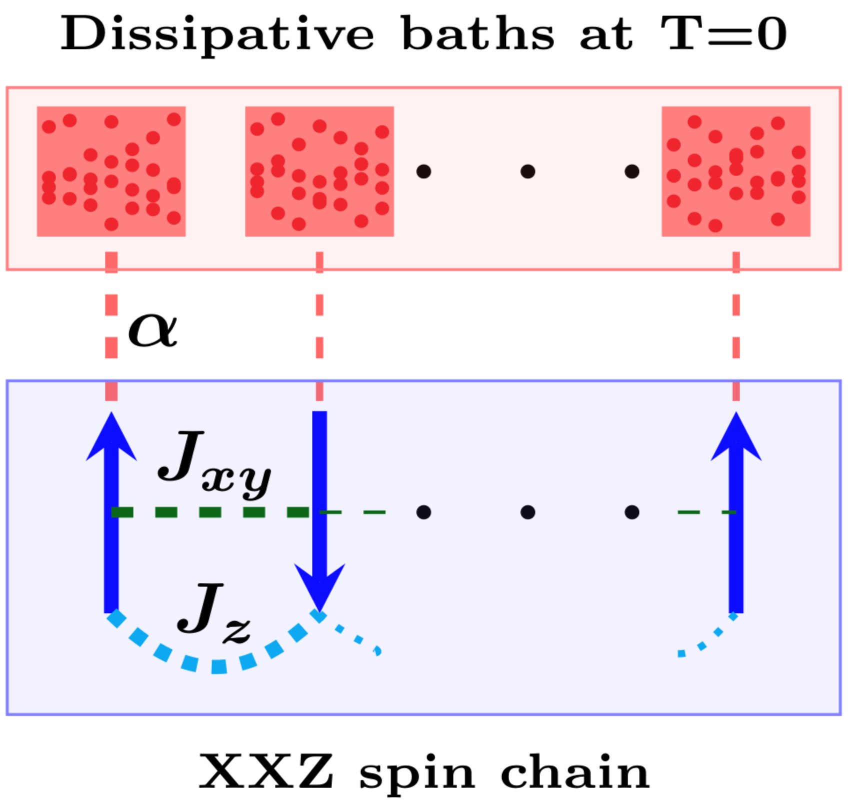
$$H = H_{\mathrm{S}} + H_{\mathrm{B}} + H_{\mathrm{SB}}$$

$$H_{\mathrm{S}} = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right)$$

$$H_{\mathrm{B}} = \sum_{jk} \frac{P_{jk}^2}{2m_k} + \frac{m_k \Omega_k^2}{2} X_{jk}^2$$

$$H_{\mathrm{SB}} = \sum_{j=1}^{N} S_j^z \sum_k \lambda_k X_{jk}$$

Schematic diagram:



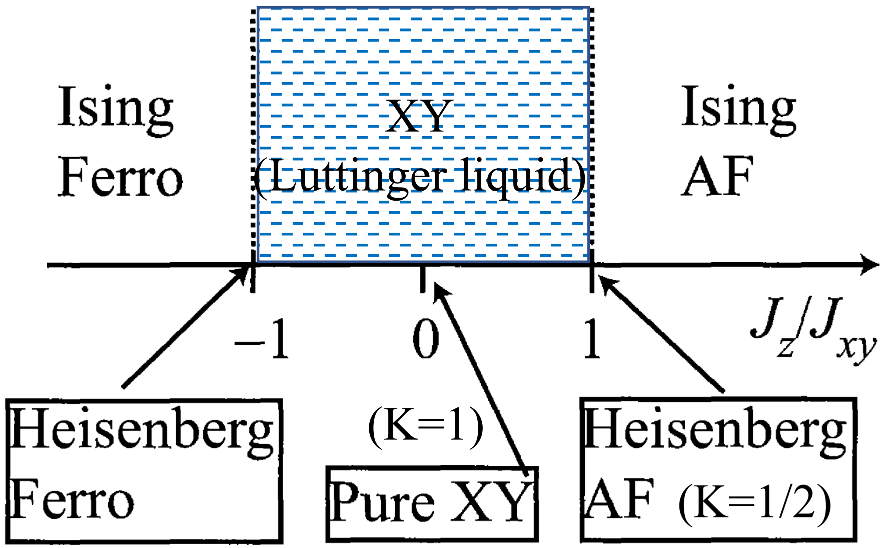
Strictly local, ohmic baths (s=1).

Last step : Path integral

Spin chain $\xrightarrow[\text{Path integral}]{\text{Bosonisation}}$ **2D statistical field theory** $\xrightarrow[\text{bath modes}]{\text{Integrate out}}$ **Effective field theory**

$$S_{\text{eff}} = S_{\text{LL}} + S_{\text{diss}}$$

$$S_{\text{LL}}[\phi] = \frac{1}{K}\int \left[\frac{1}{u}\left(\partial_\tau \phi(x,\tau)\right)^2 + u\left(\partial_x \phi(x,\tau)\right)^2\right] dx\, d\tau$$



Properties:

- Luttinger liquid.
- Gapless spectrum.
- Delocalised, perfectly conducting (metallic) phase.

Last step : Path integral

$$S_{\text{eff}} = S_{\text{LL}} + S_{\text{diss}}$$

$$J(\Omega) \approx \pi\alpha\Omega^{s}$$

$$S_{\text{diss}} = -\alpha \int \frac{\cos\left(2\left(\phi(x,\tau) - \phi(x,\tau')\right)\right)}{|\tau - \tau'|^{2}}\, dx\, d\tau\, d\tau'$$

Properties:

- Long-range in nature.
- Only along τ (imaginary time direction) – bath is local in nature!
- For a generic ohmicity s, the exponent is 1+s.
- Action valid for finite magnetic field sector.

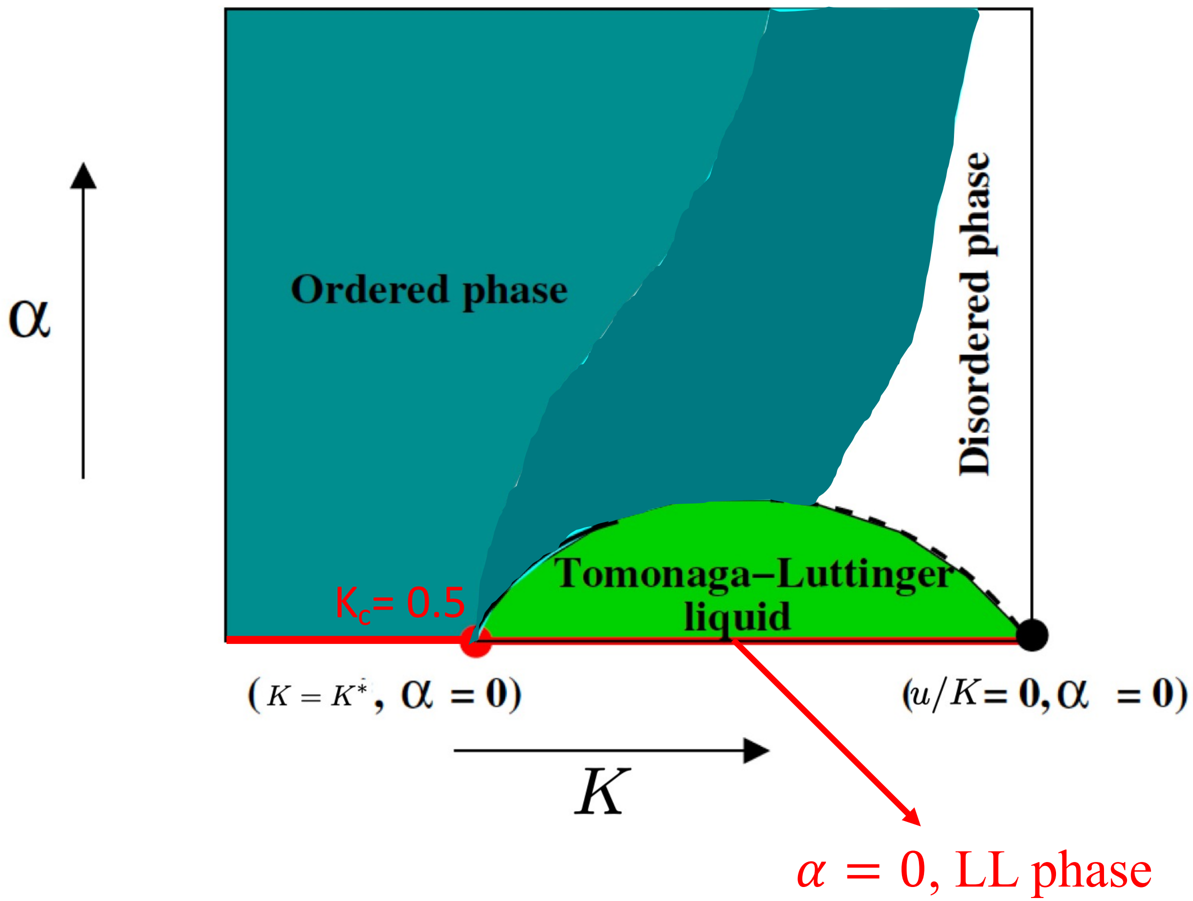
Bibliography

Cazalilla et al [PRL, 2006]:

- Perturbative RG study on the action.
- K_c= 0.5, BKT transition (z=1).

Schollwock et al [PRL, 2014]:

- Numerical study on the quantum model.
- z=2 type transition at K=1.



Approaching the problem : Numerics

Langevin dynamics :

$$\frac{d\phi_{ij}(t)}{dt} = -\frac{\delta S[\phi_{ij}(t)]}{\delta \phi_{ij}} + \eta_{ij}(t)$$ (i,j = discretised space and imaginary time, t = langevin time)

$$= \frac{u}{K\pi}\left[\phi_{i+1,j} + \phi_{i-1,j} - 2\phi_{i,j}\right] + \frac{1}{uK\pi}\left[\phi_{i,j+1} + \phi_{i,j-1} - 2\phi_{i,j}\right]$$

$$+ \frac{\alpha}{\pi^2}\sum_{j'} D(|j-j'|)\sin\left[2(\phi_{ij'} - \phi_{ij})\right] + \eta_{ij}(t)$$ (D(|j-j'|) = $1/\tau^2$ discretised kernel with PBC)

$$\langle \eta_{ij}(t)\rangle = 0 \qquad \langle \eta_{ij}(t)\eta_{i'j'}(t')\rangle = 2\delta_{i,i'}\delta_{j,j'}\delta(t-t')$$

- System size : scale β as L
- Spatially homogenous : Use u =1.
- Algorithm : Stochastic 2nd order Runge-Kutta for white noise [Honeycutt, PRA 1992]
- Generate equilibriated configurations to calculate correlation functions.

Approaching the problem : calculations

Variational ansatz : Obtain an effective quadratic action by minimising the free energy!

$$G_{\mathrm{LL}}^{-1}(q,\omega_n) = \frac{1}{2\pi K_r}\left(u_r q^2 + \frac{\omega_n^2}{u_r}\right)$$

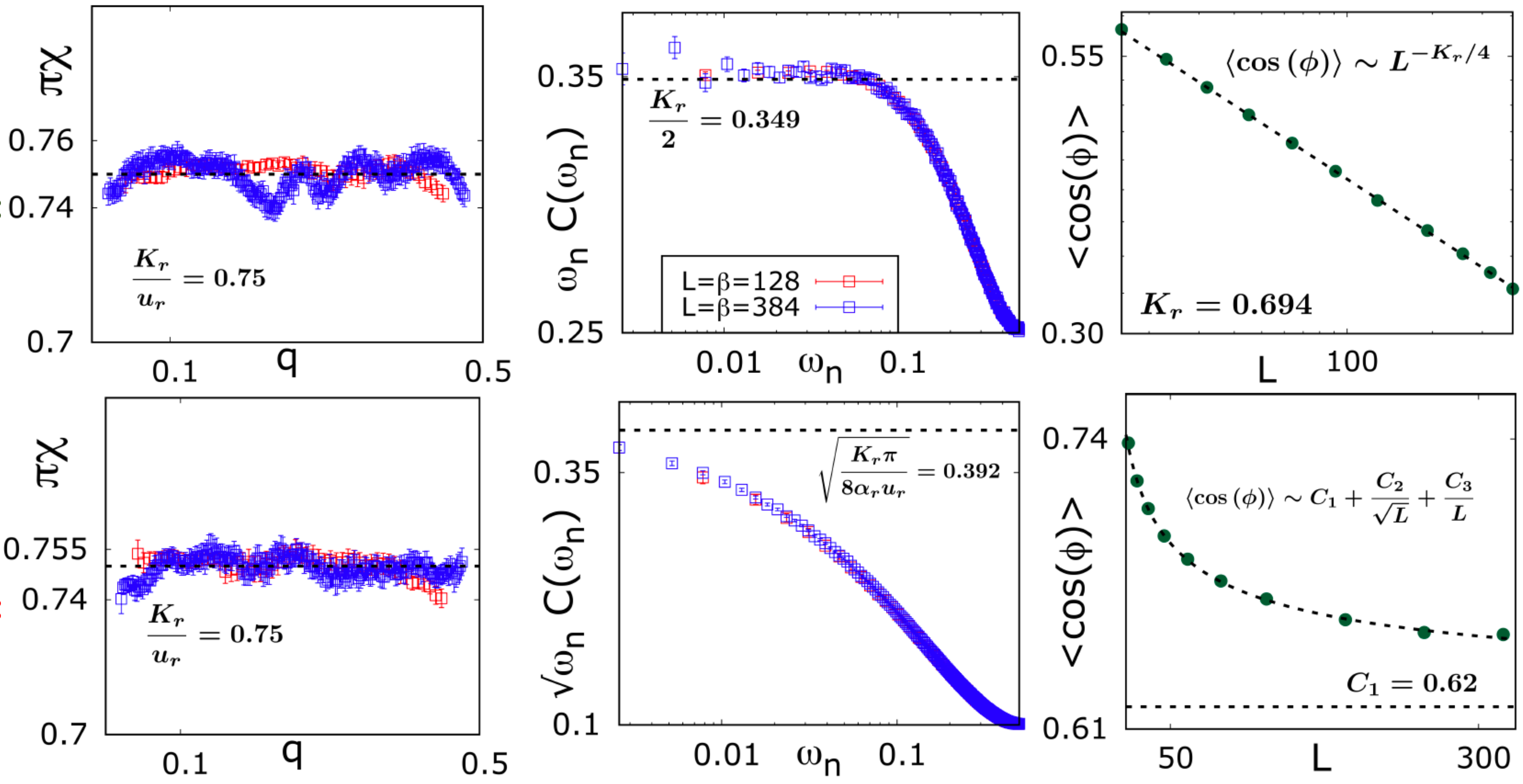
$$G_{\mathrm{ord}}^{-1}(q,\omega_n) = \frac{u_r q^2}{2\pi K_r} + \frac{\alpha_r}{\pi^2}|\omega_n| + a_1 |\omega_n|^{\frac{3}{2}} + a_2\omega_n^2$$

Observable	**Luttinger liquid($\alpha = 0$)**	**Dissipative phase($\alpha \neq 0$)**
$\chi = \lim_{q\to 0}\lim_{\omega_n\to 0}\frac{q^2}{\pi^2}\langle\phi(q,\omega_n)\phi(-q,-\omega_n)\rangle$	$K_r/(u_r\pi)$	$K_r/(u_r\pi)$
$C(\omega_n) = (1/\pi L)\sum_q\langle\lvert\phi(q,\omega_n)\rvert^2\rangle$	$K_r/2\omega_n$	$\sqrt{K_r/8\pi u_r}(\alpha_r\omega_n/\pi^2 + a_1\omega_n^{3/2} + a_2\omega_n^2)^{-1/2}$
$\langle\cos(\phi)\rangle$	$L^{-K_r/4}$	$c_1 + c_2/\sqrt{L} + c_3/L$

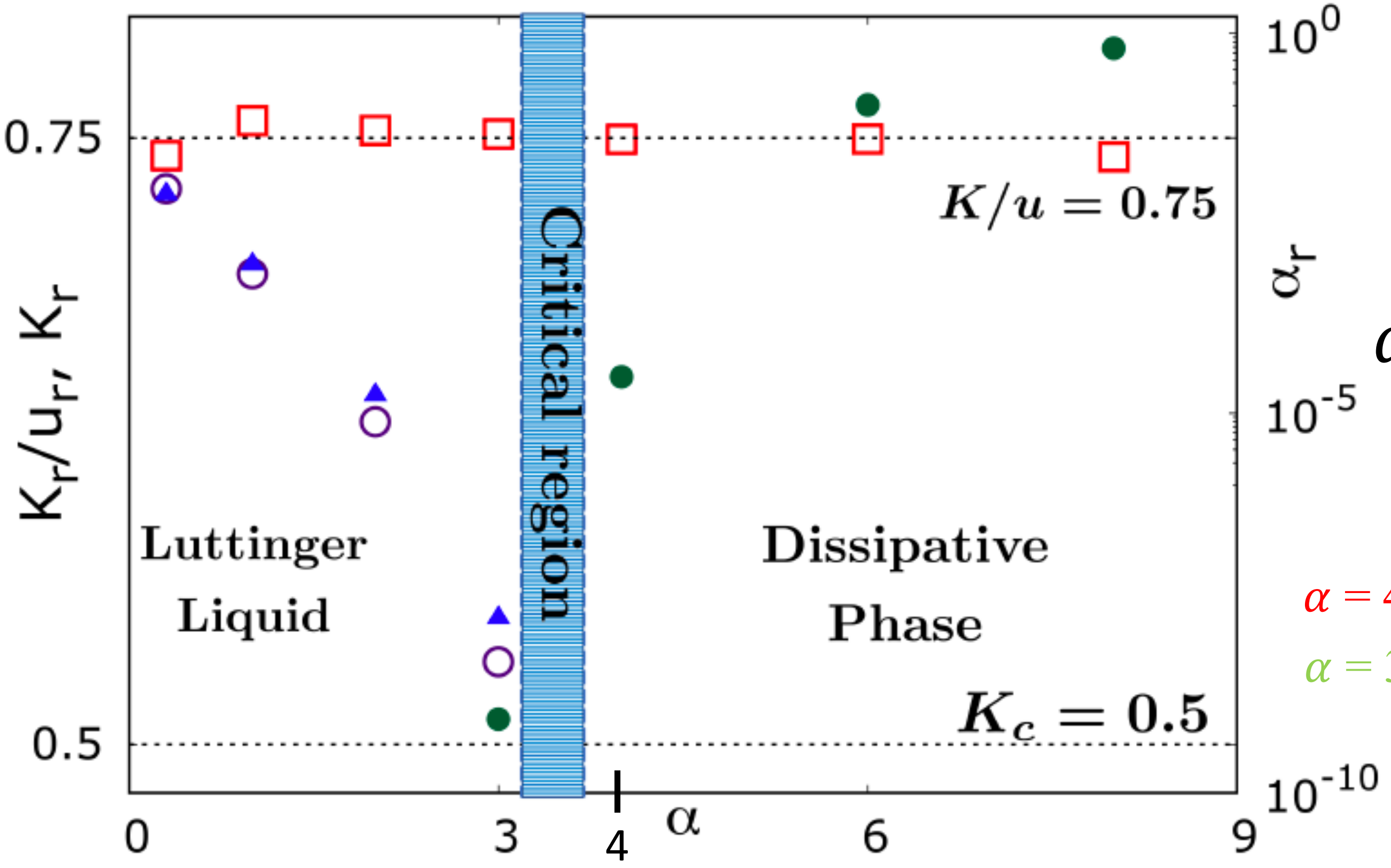
Results

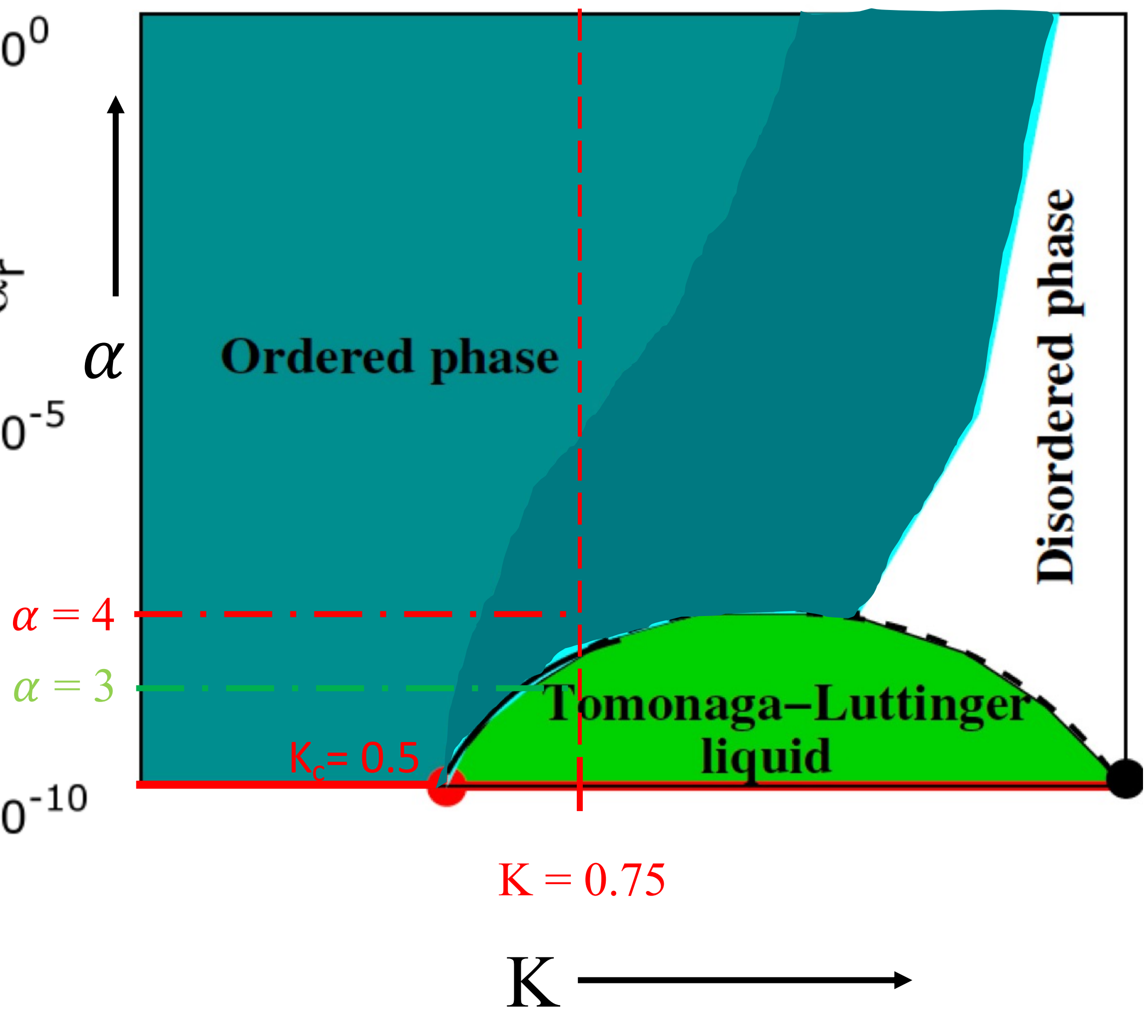
K=0.75, $\alpha = 1$:
(LL Phase)

K=0.75, $\alpha = 8$:
(Ordered Phase)



Results





- □ K_r/u_r
- ○ K_r, $<\cos(\phi)>$
- ▲ K_r, $C(\omega_n)$
- ● α_r

Transport properties

Conductivity :

$$\sigma(\omega) = \frac{e^2}{\pi^2\hbar}\left[\omega_n G(q=0,\omega_n)\right]_{i\omega_n \to \omega + i\epsilon}$$

Phase	DC conductivity (Re($\sigma(\omega \to 0)$))
LL	$(e^2 uK/\hbar)\delta(\omega)$
Ohmic bath	$e^2/\hbar\alpha_r$
Subohmic bath	0

Possibility of Localisation!

Conclusions

- In one dimensional interacting systems, local baths have an effect on the existing Luttinger liquid phase.
- For an ohmic bath, there exists a KT phase transition for K_c=0.5.
- For K > K_c, above a critical α the system enters into an ordered phase with unaltered susceptibility (compressibility) and reduced conductivity.
- For subohmic bath, there exists a possibility of the conductivity reducing to zero, which is signature of localization in the system.

Future directions

- Checking the subohmic finite magnetic field spin chain case.
- Checking the zero magnetic field spin chain case.
- Understanding the possibility of existence of a third phase ("disordered phase" of Cazalilla).

THANK YOU !

Appendix: Jordan-Wigner transformation

Spin chain —— Jordan-Wigner transf. ——→ Interacting Fermionic system

$$S_j^+ \equiv S_j^x + iS_j^y \rightarrow c_j^\dagger e^{i\pi \sum_{i<j} n_i} \qquad S_j^z \rightarrow n_j - \frac{1}{2}$$

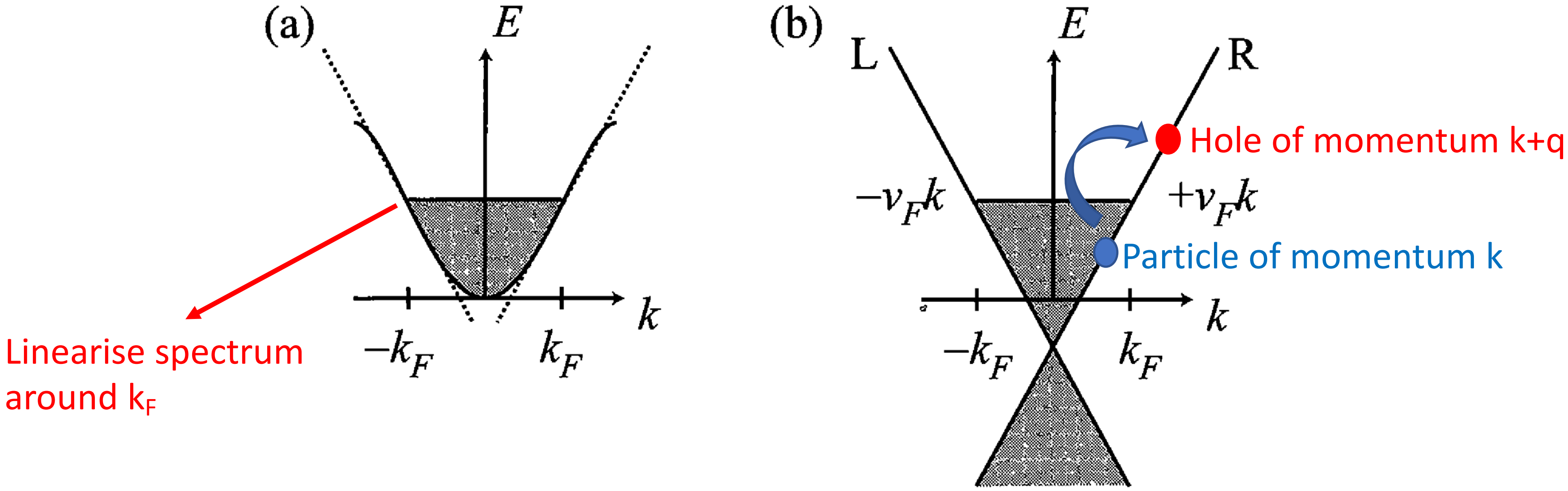
$$H_{\mathrm{S}} = \frac{J_{xy}}{2} \sum_i c_{i+1}^\dagger c_i + \mathrm{h.c} + J_z \sum_i \left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right)$$

$$H_{\mathrm{SB}} = \sum_i \left(n_i - \frac{1}{2}\right) \sum_k \lambda_k X_{ik}$$

Apendix: Bosonisation

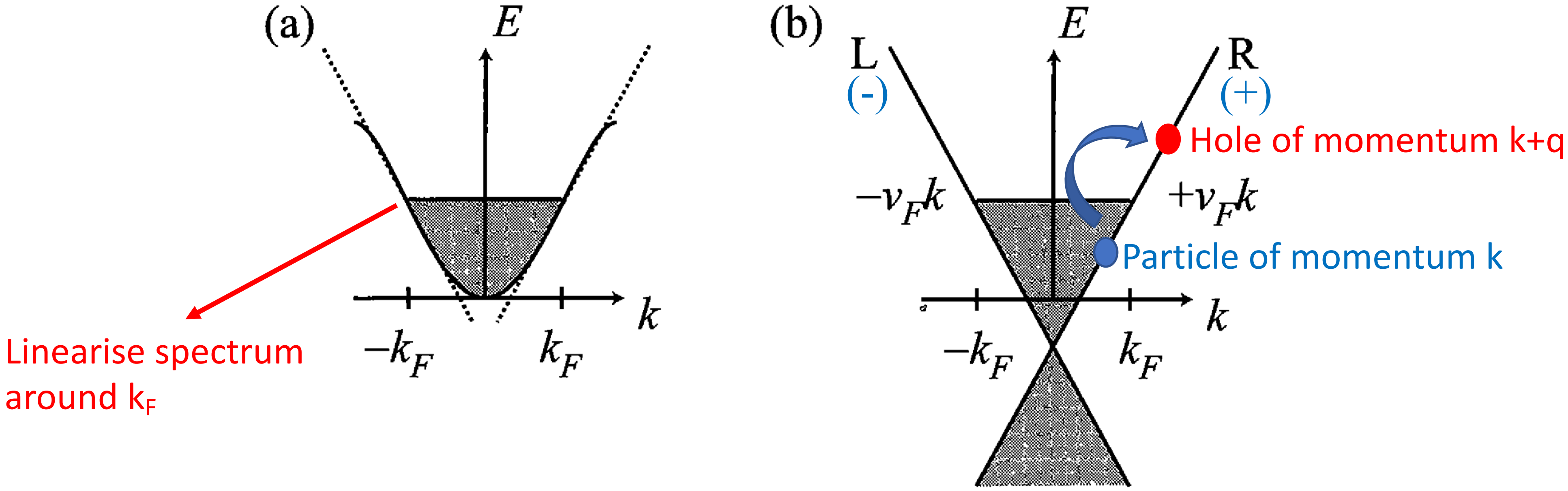
1D Fermionic system —Bosonisation→ **field theory**

- Technique specifically designed for 1D systems. [Giamarchi, Quantum Physics in one dimension]
- Captures low energy excitation physics of the system.



Simple observation : $\rho^{\dagger}(q) = \sum_{k} c^{\dagger}_{k+q} c_{k}$ is bosonic in nature.

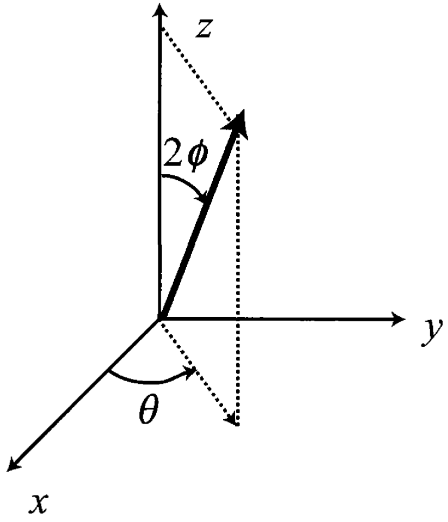
Appendix: Bosonisation (recipe)



$$1.\ c_j = \frac{1}{\sqrt{N}}\left[\sum_{k\approx k_F} e^{ikx}c_k + \sum_{k\approx -k_F} e^{ikx}c_k\right] = [\psi_+(x) + \psi_-(x)]$$

$$2.\ \psi_r(x) = \frac{1}{\sqrt{2\pi}} e^{i(r(k_F x - \phi(x)) + \theta(x))}$$

Bosonic field operators



Appendix: Variational ansatz

Variational ansatz: Obtain an effective quadratic action by opitimising the free energy [Feynmann, statistical mechanics: a set of lectures, 1998]!

Recipe :

1. Assume : $$\boldsymbol{S}_{\text{var}} = \frac{1}{2\beta L} \sum_{q,\omega_n} \phi^*(q,\omega_n) G_{\text{var}}^{-1} \phi(q,\omega_n)$$

2. Calculate : $$\boldsymbol{F}_{\text{var}} = \frac{1}{\beta} \left(- \sum_{q,\omega_n} \log G_{\text{var}} + \langle S - S_{\text{var}} \rangle_{S_{\text{var}}} \right)$$

3. Minimise : $$\frac{\partial F_{\text{var}}}{\partial G_{\text{var}}} = 0$$

Appendix : Results for K=0.55

K=0.55, $\alpha = 0.05$:

K=0.55, $\alpha = 10$:

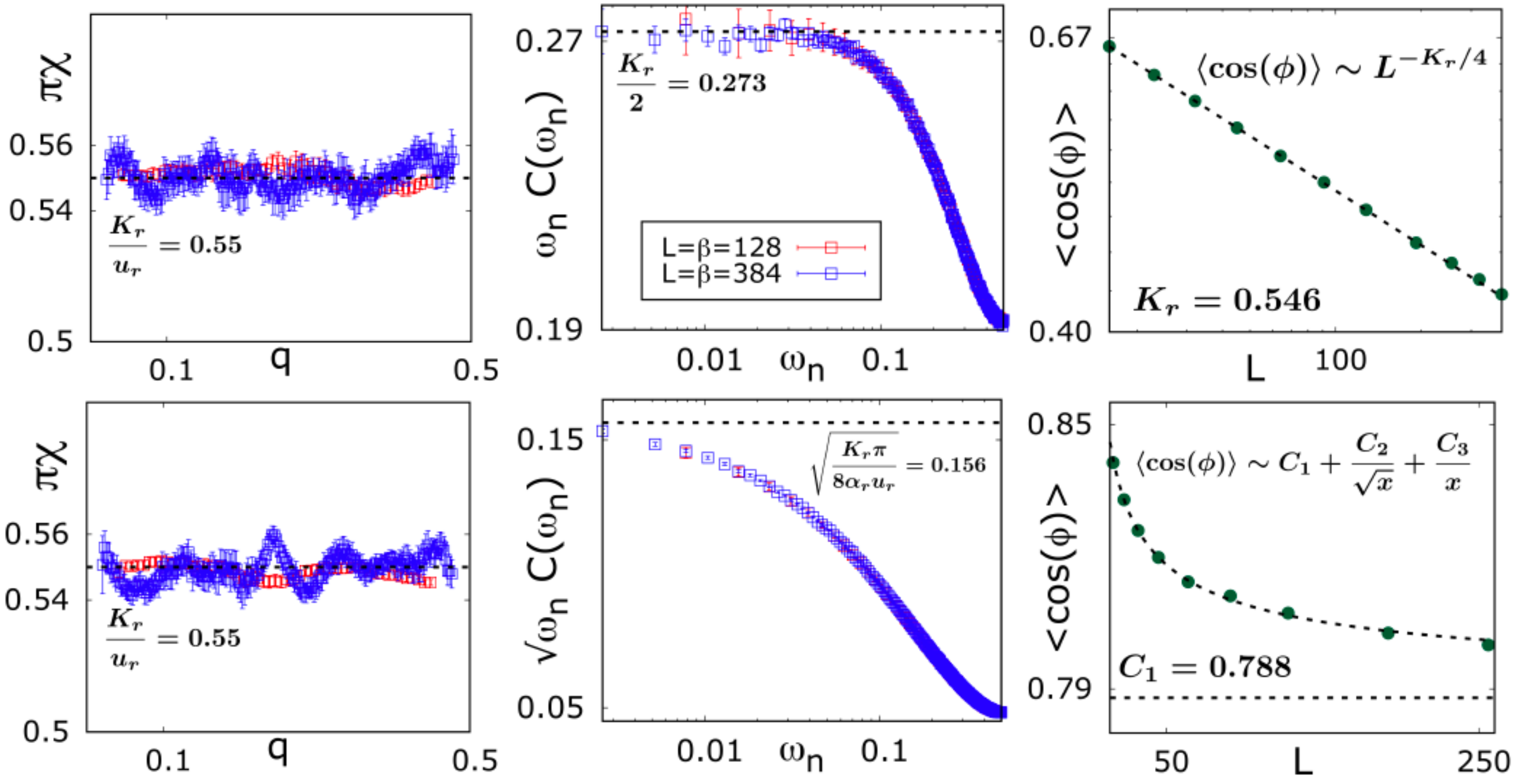


FIG. 1. Calculation of different quantities for $K = 0.55$ that characterizes LL ($\alpha = 0.05$, top row) and dissipative phase ($\alpha = 10$, bottom row). Blue and red points correspond to $L = \beta = 384$ and $L = \beta = 128$, respectively. *Left* : Due to symmetry, $\pi\chi = K_r/u_r$ is equal to $K/u = 0.55$ for all values of α and all lengthscales. *Middle* : For $\alpha = 0.05$, $\omega_n C(\omega_n)$ saturates to $K_r/2 = 0.273$ as $\omega_n \to 0$; whereas for $\alpha = 10$, $\sqrt{\omega_n} C(\omega_n)$ saturates to $[K_r\pi/(8\alpha_r u_r)]^{1/2} = 0.156$. The other fitting constants are $a_1 = 16.61$ and $a_2 = 571.4$. *Right* : For $\alpha = 0.05$, $\langle \cos(\phi) \rangle$ decays as a power law, which allows us to extract $K_r = 0.546$, consistent with the fit of $\omega_n C(\omega_n)$. For $\alpha = 10$ it saturates to a constant, as predicted by the variational ansatz (the fit gives $c_1 = 0.788$, $c_2 = 0.215$ and $c_3 = 0.012$).

Appendix : Results for K=0.55