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Laboratoire de Physique Théorique
et Modèles Statistiques

Interaction, Disorder, Elasticity GDR
GRENOBLE, Nov 2022

Quantum dynamics with disorder:
Localization, and how it is (not) related to glassiness.

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Laboratoire d'Excellence Physique : Atomes Lumière Matière

I. MBL is not a quantum Glass.

- Quantum Disordered Systems: Frameworks
- MBL: A tentative definition
  - The Ingredients
  - The Questions
  - The Picture
- MBL is not a Glass with $\hbar > 0$.

II. Localization is glassy, though.

- Computing Decay Rates:
- The directed polymer problem
- Localization is glassy, though

Quantum Disordered Systems: Frameworks

EQUILIBRIUM: $\mathcal{Z}_\beta = \mathrm{tr}\{e^{-\beta\hat{H}}\}$

OPEN SYSTEMS

ISOLATED SYSTEMS

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quantum disordered system + bath

- (Quantum) phase transitions
- Langevin dynamics

e.g. Bose glass [GIAMARCHI, SCHULZ], [TALK THIBAUD M.]

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quantum disordered system + driving + bath

- stationary states
- interplay bath-driving

eg. [MAIMBOURG, ROSSO ET AL.]
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ISOLATED SYSTEMS

quantum disordered system with unitary dynamics

- integrable systems
- Many-Body localization (MBL)

[THIS TALK]

MBL: A tentative definition

" MBL systems are thermodynamically-large, INTERACTING systems (spins, cold atoms, qbits, electrons...) in QUENCHED DISORDER, that evolve with QUANTUM UNITARY DYNAMICS and never reach thermal equilibrium because transport is SUPPRESSED. "

Seminal works:

[ANDERSON 1958]

[ALTSHULER, GEFEN, KAMENEV, LEVITOV 1997]

[BASKO, ALEINER, ALTSHULER 2005]

Reviews:

[ABANIN, ALTMAN, BLOCH, SERBYN 2019 - REV. MOD. PHYS. 91]

[ANNALEN DER PHYSIK: VOL 529, NO 7]

The Ingredients

■ **UNITARY EVOLUTION:** at $t=0$, pure state: $|\psi(0)\rangle$

at $t \geq 0$, $|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$, $\hat{U}(t) = e^{-it\hat{H}}$ ← Hamiltonian

density matrix $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$

Observable $\hat{O}$: $\langle\hat{O}\rangle(t) = \mathrm{tr}\{\hat{O}\,\hat{\rho}(t)\}$

Preserves: energy $E = \langle\psi(0)|\hat{H}|\psi(0)\rangle$ – time independence of $\hat{H}$

"purity" of state: $\mathrm{tr}[\hat{\rho}^2(t)] = 1$ – Unitarity

memory of initial condition – reversible dynamics

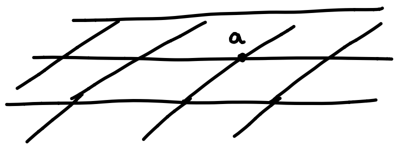
NO BATH, NO LANGEVIN, NO DRIVING, NO DISSIPATION!

QUENCHED RANDOMNESS:

$$\hat{H}_{And} = W \sum_a \varepsilon_a \hat{n}_a - J \sum_{\langle a,b \rangle} \left(\hat{c}_a^+ \hat{c}_b + \hat{c}_b^+ \hat{c}_a \right)$$

Kinetic energy

random fields $\{\varepsilon_a\}$ iid with given $p(\varepsilon)$



$\hat{c}_a^+$ = creates fermion at site a

$\hat{n}_a = \hat{c}_a^+ c_a$ = counts fermions at site a.

LOCAL INTERACTIONS:

$$\hat{H}_{MBL} = \hat{H}_{And} + U \sum_{\langle a,b \rangle} \hat{n}_a \hat{n}_b$$

interactions short-ranged in space

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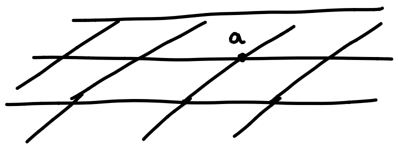

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Notice: fermionic model in 1d is equivalent to spin-chain:

$$\hat{H}_{XXZ} = W \sum_a \varepsilon_a \hat{\sigma}_a^z - J \sum_{\langle a,b \rangle} \left(\hat{\sigma}_a^x \hat{\sigma}_b^x + \hat{\sigma}_a^y \hat{\sigma}_b^y \right) + U \sum_{\langle a,b \rangle} \hat{\sigma}_a^z \hat{\sigma}_b^z$$

with $\hat{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\hat{\sigma}^y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $\hat{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(more handy for numerics)

The Questions

I. Transport and Decay

- DECAY RATES OF LOCAL EXCITATIONS: Create local excitation $\hat{c}_a^+$ (a = site) on top of many-body state $|\psi_{MB}\rangle$ of (interacting) particles: $|\psi(0)\rangle = \hat{c}_a^+|\psi_{MB}\rangle$
Does it decay (↔ spread out in system), or local memory?

- TRANSPORT OF CONSERVED QUANTITIES (e.g. energy): even-out imbalance with diffusive modes?

[ANDERSON: "Absence of diffusion in certain random lattices"]

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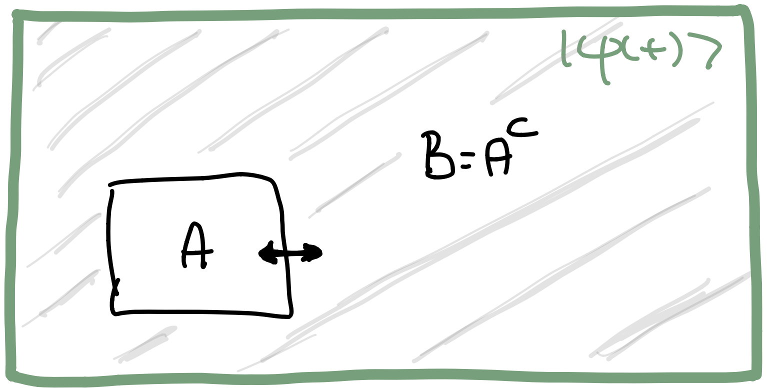
II. Growth of quantum correlations

- ENTANGLEMENT ENTROPY: $S_{vN}(t) = -\mathrm{tr}\{\hat{\rho}(t)\log\hat{\rho}(t)\}$ Vanishes for pure state $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$
  (stays zero under unitary dynamics)

SUBSYSTEMS entanglement entropy:



At $t=0$: $|\psi(0)\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$

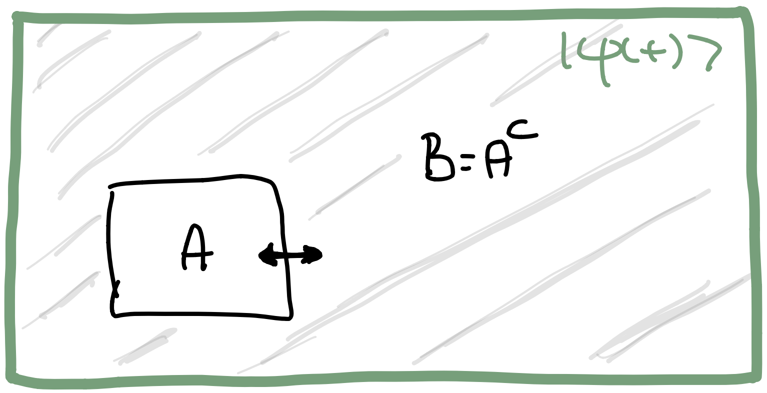
Reduced density matrix:

$$\hat{\rho}_A(t) = \mathrm{Tr}_B\{\hat{\rho}(t)\} = \mathrm{tr}_B\{|\psi(t)\rangle\langle\psi(t)|\}$$

$$S_{AB}(t) = -\mathrm{tr}\{\hat{\rho}_A(t)\log\hat{\rho}_A(t)\}$$

If system uncoupled, $S_{AB}(t) = 0 \quad \forall t$.

- Subsystems entanglement entropy:



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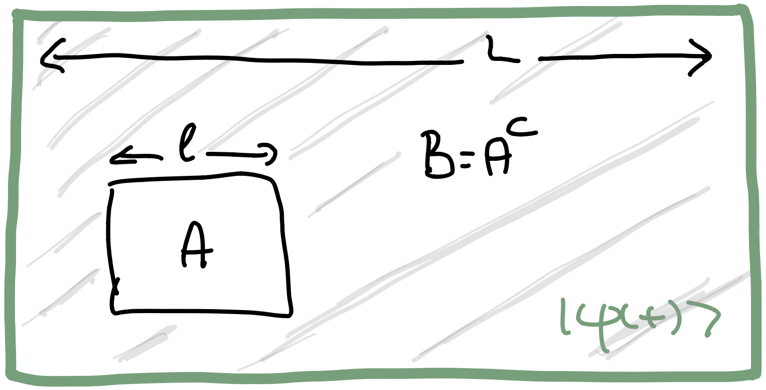
III. Emergent Thermal Equilibrium

- "Naive" notion of thermalization: $\underbrace{\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|}_{\text{pure state}} \xrightarrow{t\to\infty,\, L\to\infty} \underbrace{\hat{\rho}_{\beta_{eff}} = \frac{1}{Z_{\beta_{eff}}} e^{-\beta_{eff}\hat{H}}}_{\text{"mixture": } \mathrm{tr}[\hat{\rho}_\beta^2] < 1}$

Impossible! dynamics preserves purity (unitarity) and memory of $|\psi(0)\rangle$ (reversibility)

Better notion: thermalization of local subsystems



$\hat{O}_A$ = local observable (operator) in A.
e.g. $\hat{n}_a$ for site $a \in A$

Thermal expectation value:

$$\mathbb{E}_\beta[\hat{O}_A] = \frac{1}{Z_\beta} \operatorname{tr}\left\{ e^{-\beta \hat{H}} \hat{O}_A \right\}$$

Thermalization if: $\lim_{\substack{t \to \pm\infty \\ L \to \infty \\ (\ell \text{ fixed})}} \operatorname{tr}\left\{ \hat{O}_A \hat{\rho}(t) \right\} = \mathbb{E}_\beta[\hat{O}_A]$

with β fixed by $\langle \psi(t) | \hat{H} | \psi(t) \rangle = E = \mathbb{E}_\beta[\hat{H}]$

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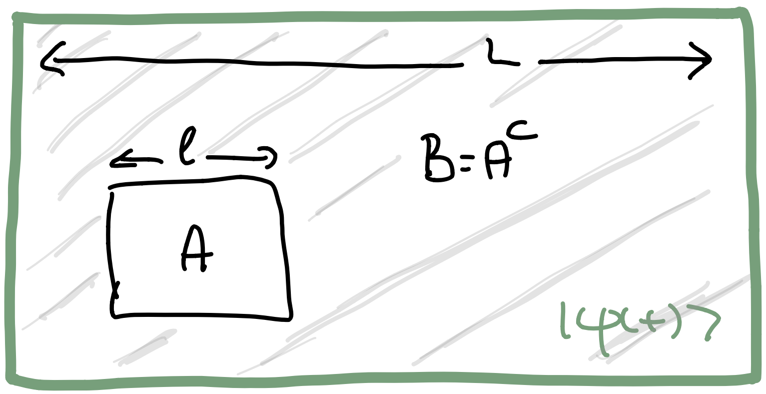

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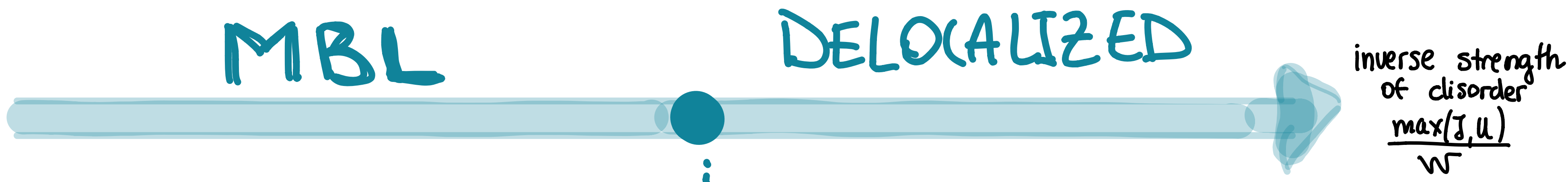
- Notice: most of these questions can be formulated as properties of the eigenstates / eigenspectrum: $\hat{H}_{MB} |\psi_\alpha\rangle = E_\alpha |\psi_\alpha\rangle \quad \alpha = 1, \dots, 2^{L^d}$
(e.g. ETH = eigenstate thermalization hypothesis)

The Picture

— from numerics on small system-sizes, approximate analytics, very few rigorous results...

$$\hat{H}_{MB} = W\sum_{a} \varepsilon_a \hat{n}_a - J\sum_{\langle a,b\rangle}\left(\hat{c}_a^{+}\hat{c}_b + \hat{c}_b^{+}\hat{c}_a\right) + U\sum_{\langle a,b\rangle}\hat{n}_a\hat{n}_b$$



Local excitations do not decay (local memory)	v.s.	Excitations have decay rate $\Gamma > 0$.
Vanishing transport coefficients $\sigma_{dc}, D = 0$	v.s.	Diffusion (or superdiffusion)
Slow growth of quantum correlations $S_{AB}(t) \sim \log t$	v.s.	Fast entanglement growth $S_{AB}(t) \sim t$
Local observables do not converge to thermal expectation value	v.s.	The system looks locally like a thermal mixture.

CAN NOT USE STATISTICAL ENSEMBLES
TO CHARACTERIZE SYSTEM AT $t \gg 1$.

DYNAMICAL TRANSITION: NO THEORY!

MBL is not a $\hbar \to 0$ Glass

- Phase is dynamical, not thermodynamical – not related to structure of Boltzmann measure, RSB etc.

  [In the language of phase transition, MBL is the unbroken-symmetry phase, symmetry is TIME REVERSAL. Order parameter: distribution of a function.]

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- MBL is not a metastable state: not due to (free-) energy barriers slowing-down equilibration. Rather, due to presence of local conservation laws. [IMBRIE, ROS, SCARDICCHIO 2017]

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- Not restricted to low energy of many-body state (i.e., ground state, $T=0$): quantumness matters at $E \gg E_{gs}$ here.

- In a glass, diffusion is there. Transport properties are different. Quantum localization.

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Computing Decay Rates: the simplest example

- Anderson problem: a single particle on a lattice, at a site "a" at $t=0$.

$$H_{And} = W\sum_a \varepsilon_a \hat{n}_a - J\sum_{\langle a,b\rangle} c_a^\dagger c_b + c_b^\dagger c_a \iff \hat{H}^{(1)} = W\sum_a \varepsilon_a |a\rangle\langle a| - J\sum_{\langle a,b\rangle}\left(|a\rangle\langle b| + |b\rangle\langle a|\right)$$

$|a\rangle$ = wave function localized at site "a": $|\psi(0)\rangle = |a\rangle$

- Localization is encoded in properties of the LOCAL SELF ENERGIES $S_a(z)$:

$$\langle a|\underbrace{\frac{1}{z-\hat{H}^{(1)}}}_{\text{propagator}}|a\rangle = \frac{1}{z - W\varepsilon_a - S_a(z)}$$

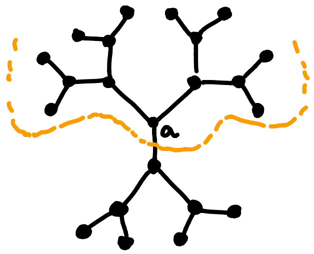
↑ shift due to presence of J

The decay rate of particle at "a" at $t=0$ is $\propto \lim_{\eta\downarrow 0} \mathrm{Im}\{S_a(E+i\eta)\}$

In the localized phase, $\lim_{\eta\downarrow 0} \mathrm{Im}\{S_a(E+i\eta)\} = 0$ IN DISTRIBUTION.

Distribution of local self-energies is hard to compute.

On Bethe lattice, cavity equations for cavity self-energies $S_a^{cav}(z)$:



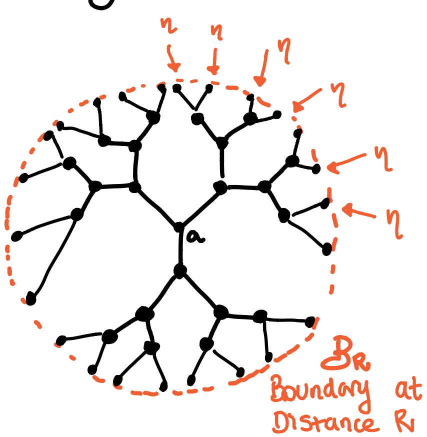
$$S_a(z) = \frac{1}{z - \varepsilon_a - \sum_{u=1}^{K-1} S_u^{cav}(z)} \qquad (*)$$

[ABOU-CHACRA, ANDERSON, THOULESS 1973]

Self-consistent description of localized phase:

(i) assume localization of isolated system: $\mathrm{Im}\{S_a^{cav}(E)\} = 0$

(ii) open system at boundary: infinitesimal decay rate $\eta \ll 1$



(iii) Assume localization: η induces <u>small</u> decay rate in bulk $\left(\mathrm{Im}\, S_a^{cav} \ll 1\right)$: linearize $(*)$

$$\mathrm{Im}\{S_a\} = \underbrace{\sum_{\mathcal{P}: a \to B_R} \prod_{s \in \mathcal{P}} \frac{J^2}{[E - \varepsilon_s - \mathrm{Re}\, S_s^{cav}(E)]^2}}_{\text{response } \Gamma} \cdot \eta$$

■ Localization: $\Gamma^{1}(E) \equiv \sum_{\mathcal{P}: a \to B_R} \prod_{s \in \mathcal{P}} \frac{J^2}{[E - W\varepsilon_s - Re\, S_a^{cav}(E)]^2} \underset{\text{value}}{\overset{\text{typical}}{\sim}} e^{-R\gamma(W,J;E) + o(R)}$

Localization stable when $\gamma(W,J;E) > 0$.

The directed polymer problem

■ Directed polymer on Bethe lattice: $Z_\beta = \sum_{\substack{\text{paths } \mathcal{P} \\ |\mathcal{P}| = R}} \prod_{s \in \mathcal{P}} e^{-\beta \varepsilon_s} \sim e^{-R f_\beta}$

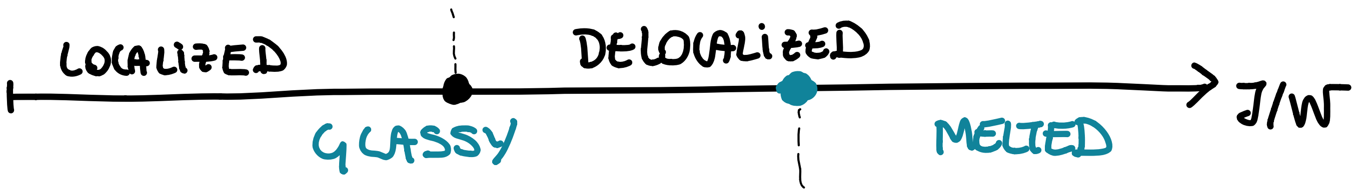
This problem has a glass transition at β_c:

$\beta < \beta_c$: Z_β contributed by $\mathcal{N} = e^{RS + o(R)}$ paths. Positive entropy $S > 0$.

$\beta > \beta_c$: Freezing: $S = 0$. Boltzmann measure condenses on sub-exponential number of paths. Replica Symmetry Breaking.

Localization is glassy, though

- When the system is localized, underlying polymer is in which phase?
  For single particles: Localization is always in frozen phase (glassy).
  System is glassy up to the transition point to delocalization.



- In interacting systems with eterogeneous (slow/fast) d.o.f, can have localized regimes that are not frozen

Interplay between localization & glassyness is rich!

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