What is the appropriate theory?

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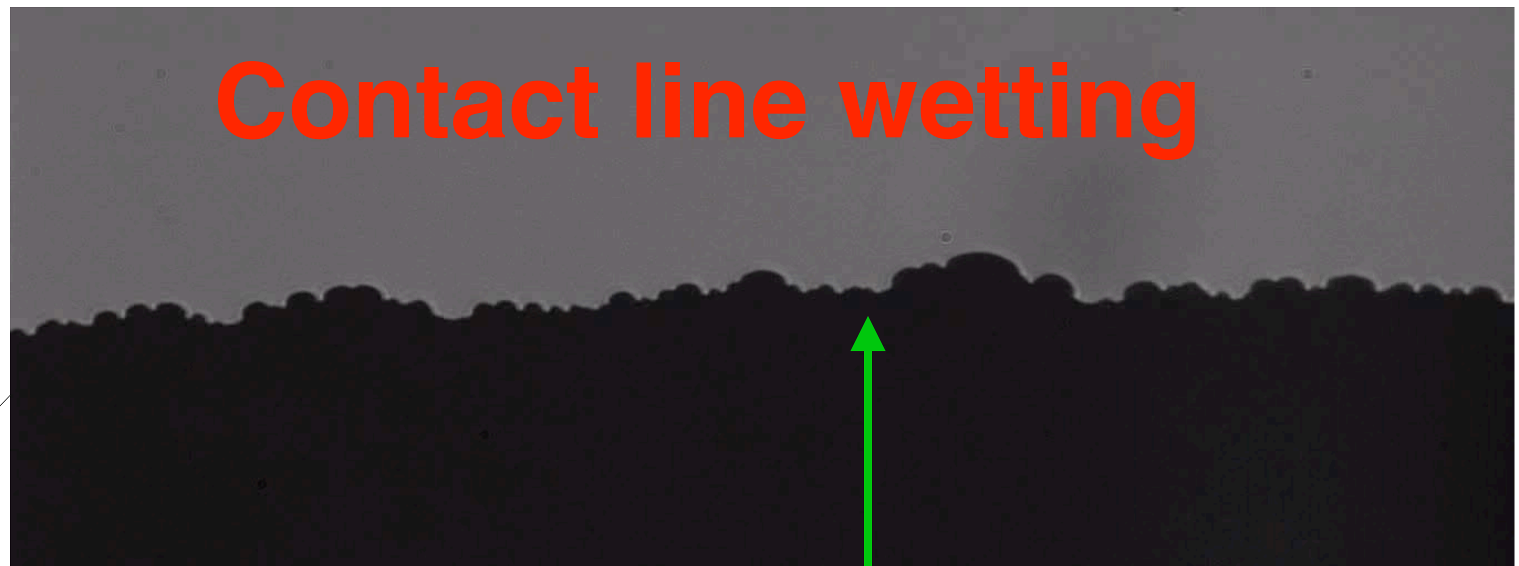
Review: arXiv:2102.01215

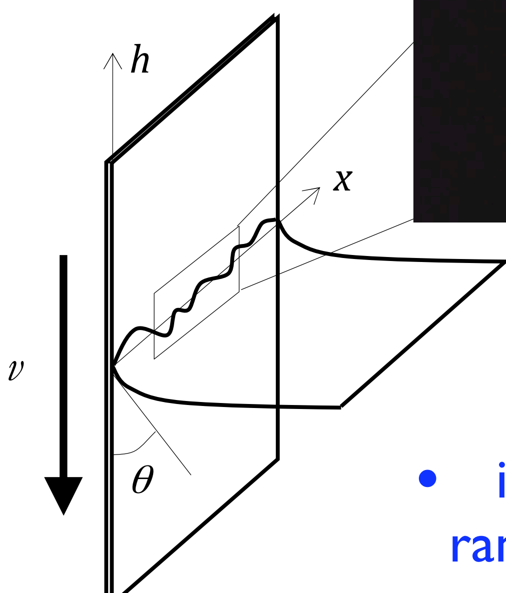
How to find the appropriate theory?

- no clue what the theory is?
- try to measure it!
- here: disordered elastic systems

(C) E. Rolley

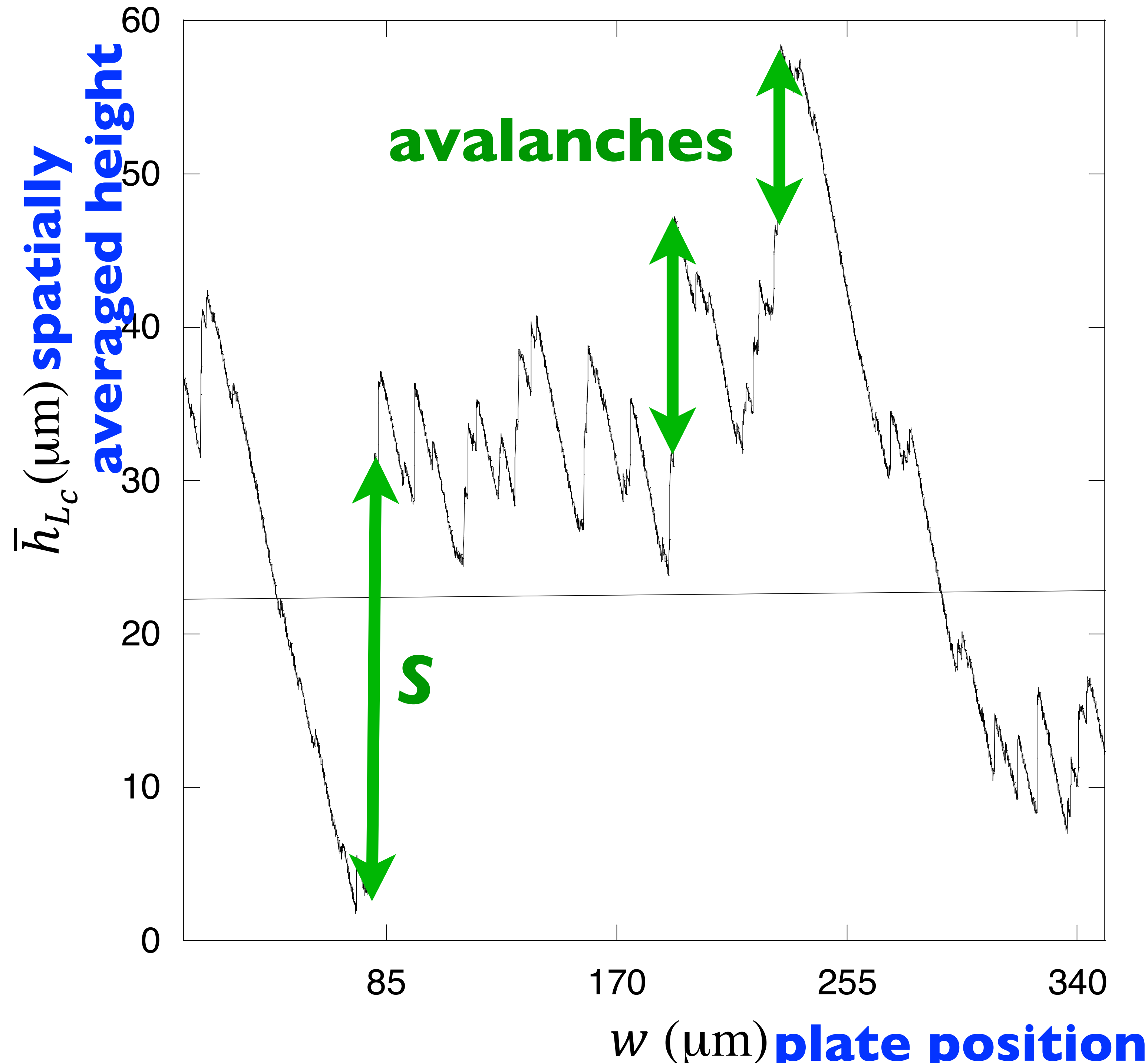


avalanche



- isobutanol on a randomly silanized

height jumps = avalanches



Theory

Equation of motion (for SR elasticity for simplicity)

height of the interface $w = vt$

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + m^2[w - u(x,t)] + F(x,u(x,t))$$

Forces are drawn from a **Gaussian**, and have correlations

$$\overline{F(x,u)F(x',u')}^{\mathrm{c}} = \delta^d(x-x')\Delta(u-u')$$

Field theory (MSR=classical limit $\hbar \to 0$ of Keldysh)

$$\mathcal{S}[\tilde{u},u] = \int_{x,t} \tilde{u}(x,t)\left[\eta\partial_t u(x,t) - c\nabla^2 u(x,t) + m^2\Big(u(x,t) - w\Big)\right] - \frac{1}{2}\int_{x,t,t'} \tilde{u}(x,t)\tilde{u}(x,t')\Delta\Big(u(x,t) - u(x,t')\Big)$$

will be measured

Why did we measure Δ?

action

unrenormalized

$$\mathcal{S}[\tilde{u}, u] = \int_{x,t} \tilde{u}(x,t)\left[\eta\partial_t u(x,t) - c\nabla^2 u(x,t) + m^2\Big(u(x,t) - w\Big)\right]$$

$$-\frac{1}{2}\int_{x,t,t'} \tilde{u}(x,t)\tilde{u}(x,t')\Delta\Big(u(x,t) - u(x,t')\Big)$$

IR scale

want to measure

external field

$$u_w := \lim_{t\to\infty} \frac{1}{L^d}\int_x u(x,t)\Big|_w$$

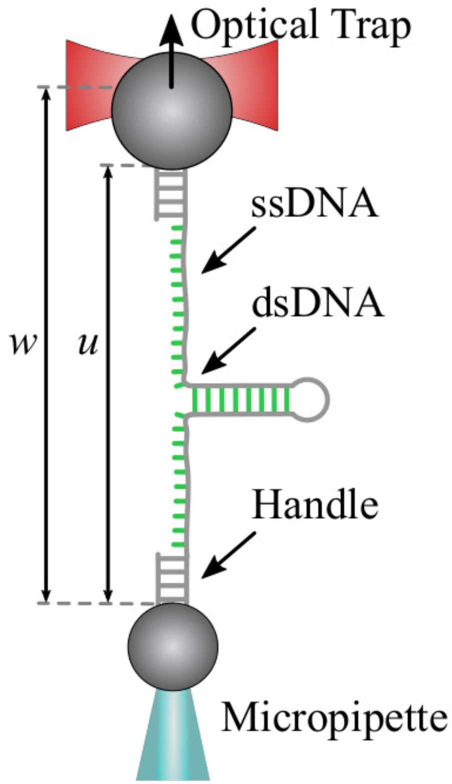
center of mass at large t, i.e. $\omega \to 0$

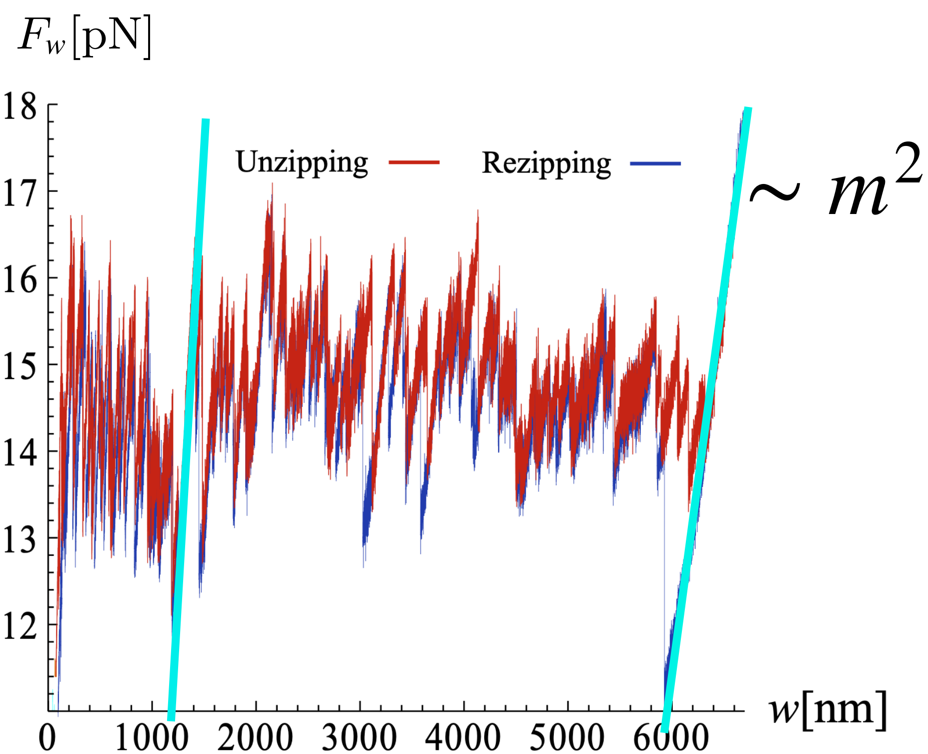
$$\Delta(w-w') \equiv \Gamma^{(2)} = \mathcal{L} \circ \overline{u_w u_{w'}}^{c} = \left[\mathcal{R}^{-1}\right]^2 \overline{u_w u_{w'}}^{c} = (m^2)^2 \overline{u_w u_{w'}}^{c}$$

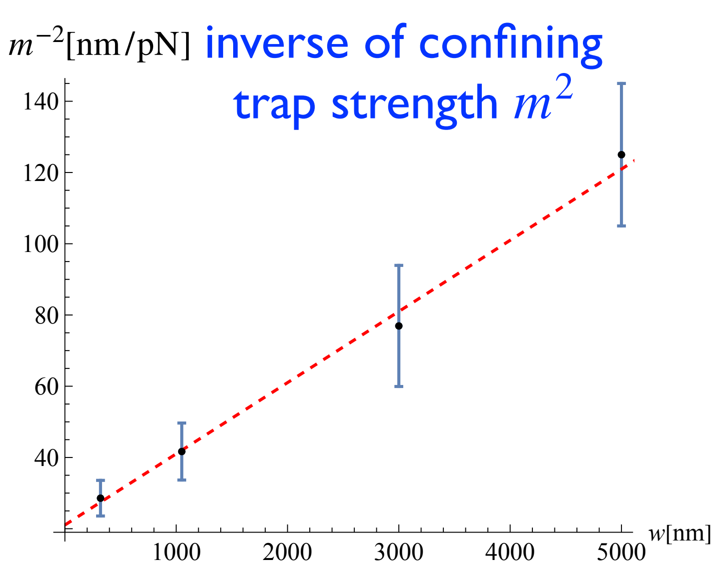
Legendre transform

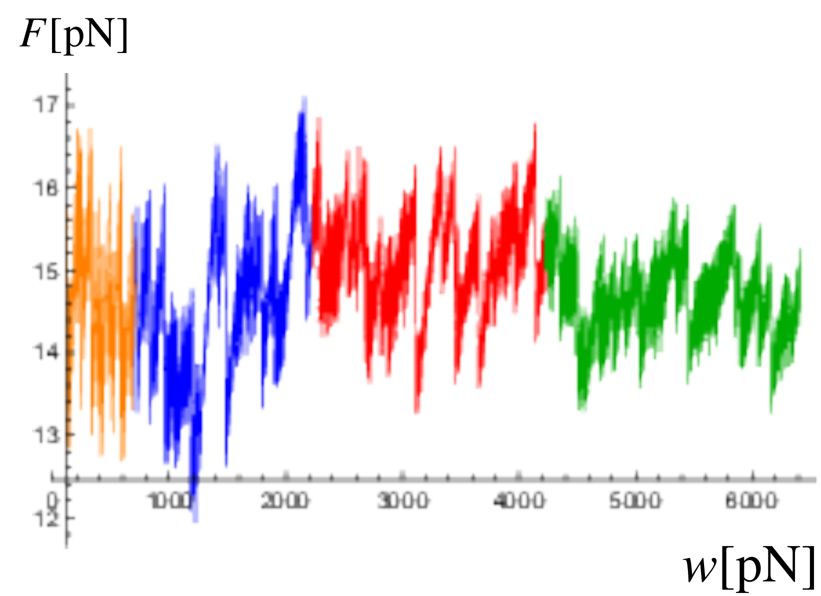
amputate 2-point function (response)

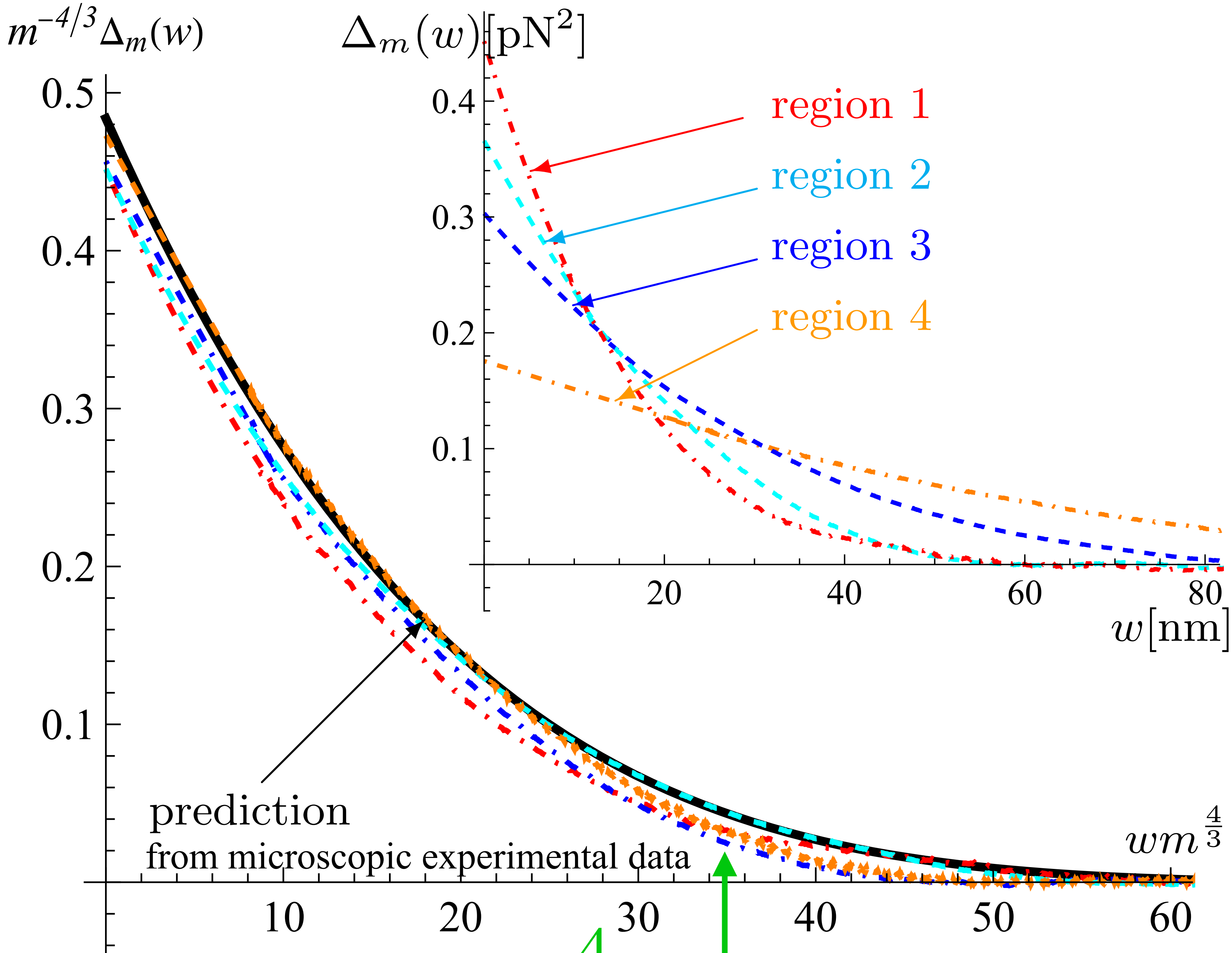
Renormalization in DNA-unzipping





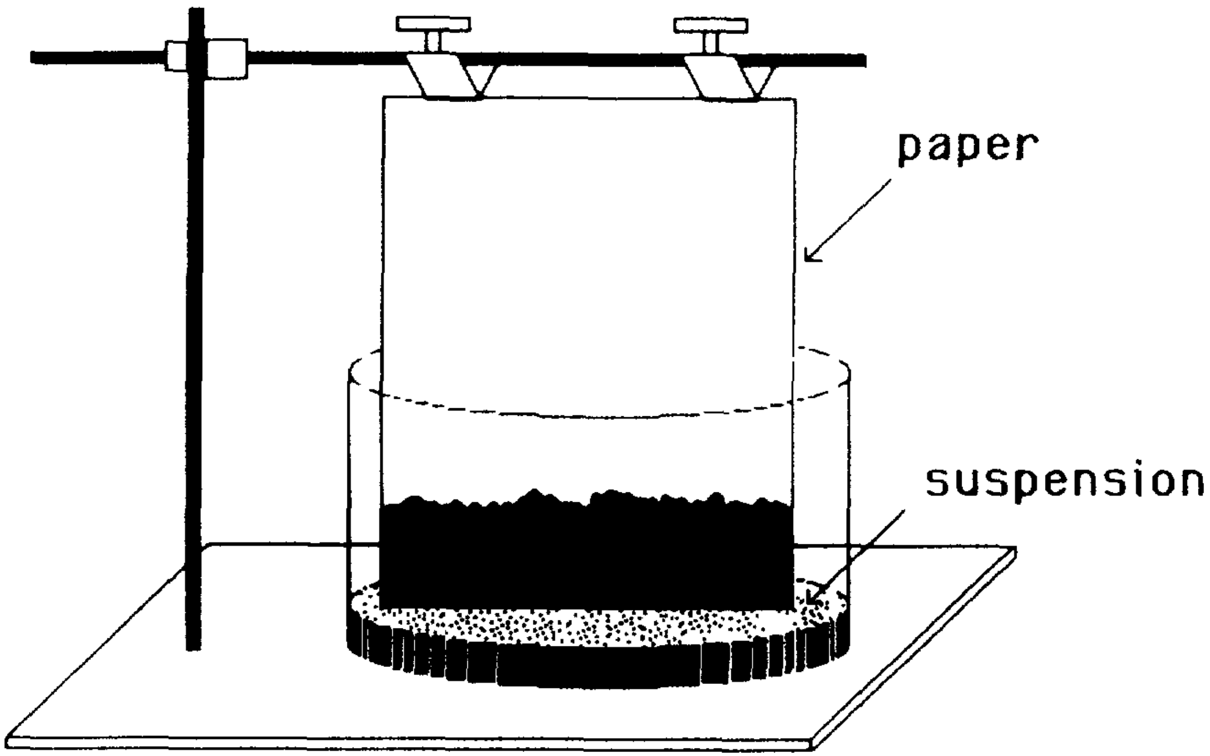






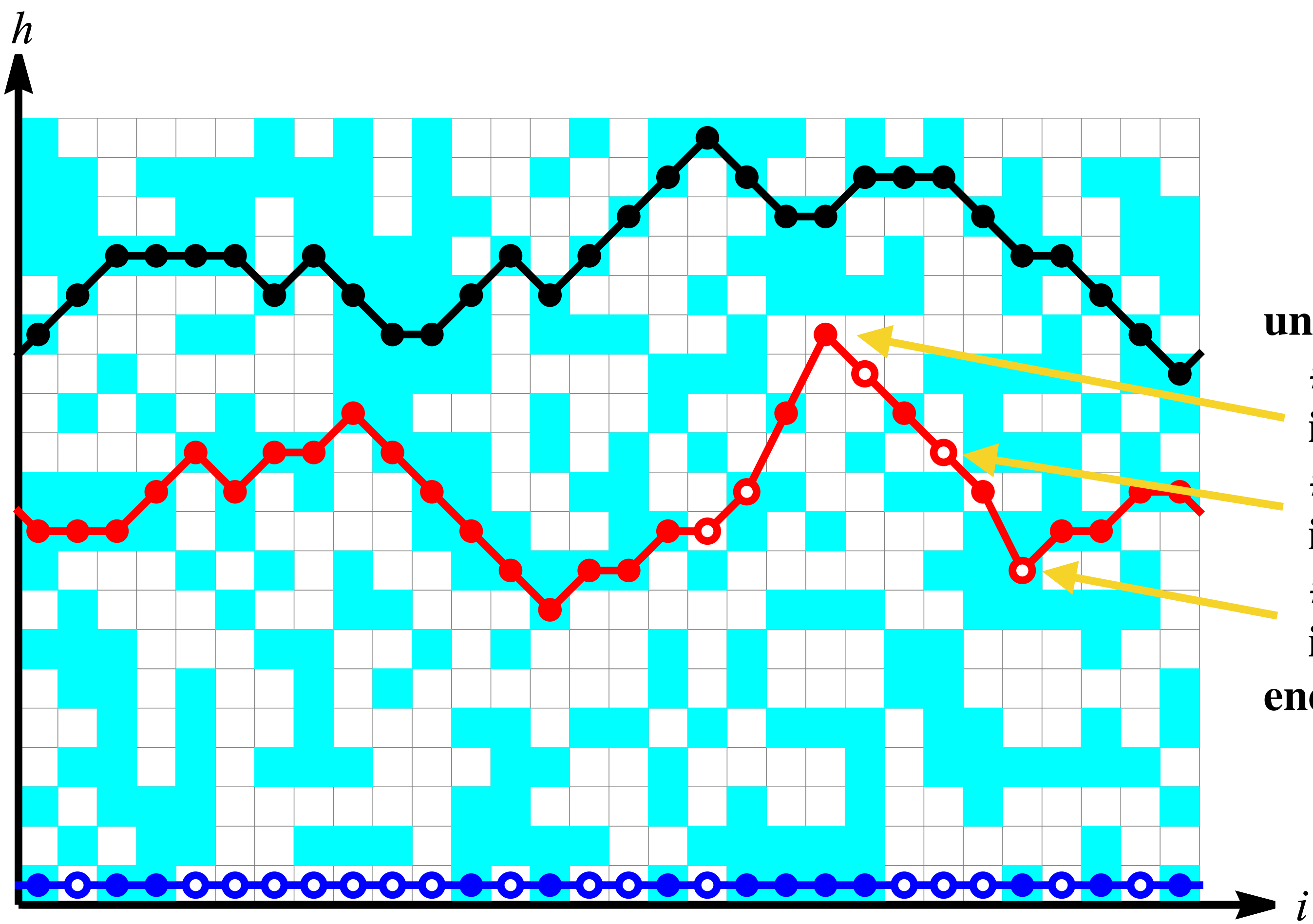
scaling collapse with $\zeta = \frac{4}{3}$

Imbibition



S.V. Buldyrev, et al., Phys. Rev. A 45 (1992) R8313–16.

The Tang-Leschhorn cellular automaton of 1992
TL92



unstable(i)

 # links cannot be longer than 2

if $h(i) - h(\text{neighbor}) \geq 2$ **return** false

 # move forward if open

if $f\big(i, h(i)\big) > f_c$ **return** true

 # move forward if a neighbor is 2 ahead

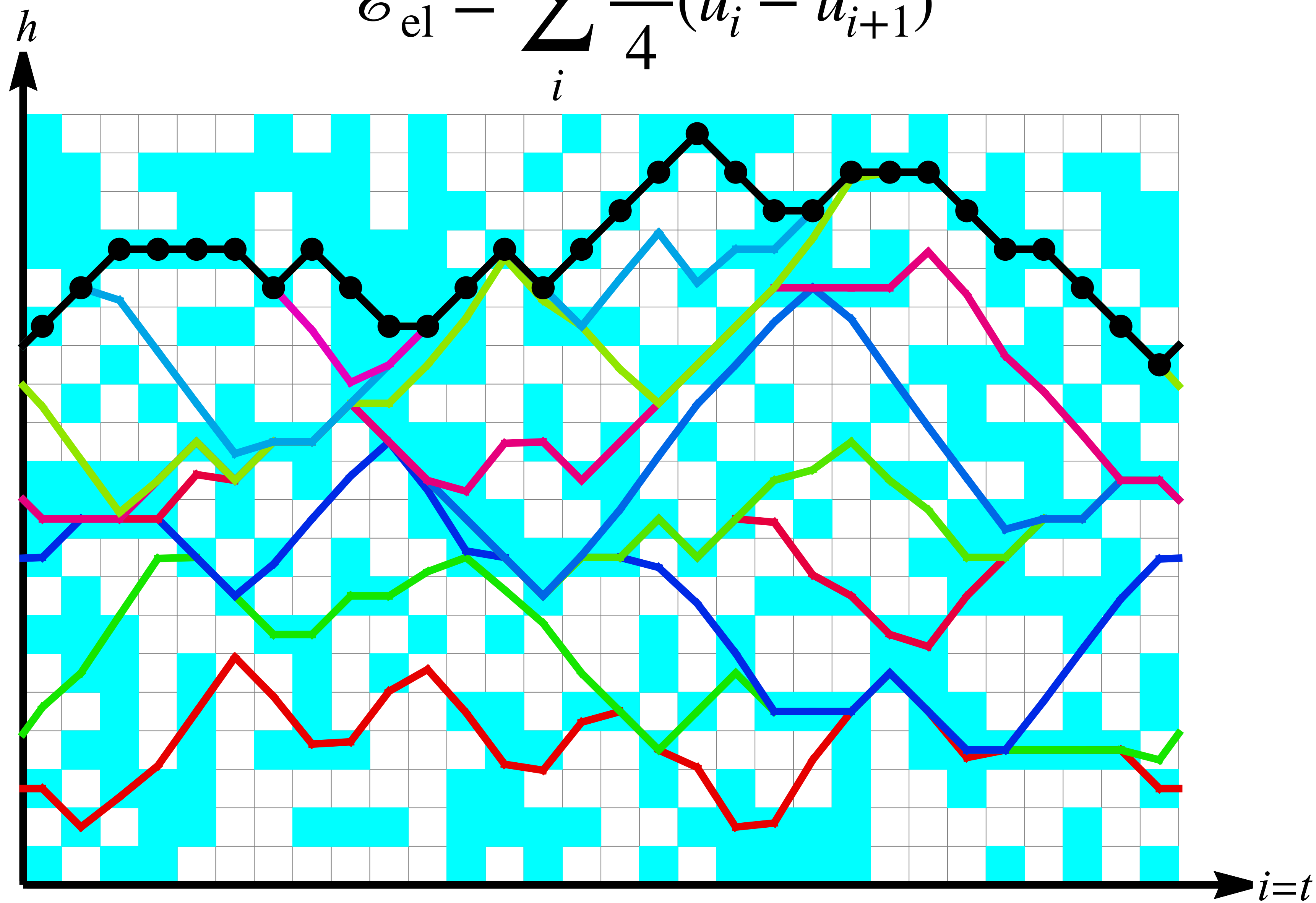
if $h(\text{neighbor}) - h(i) \geq 2$ **return** true

end

variants: Buldyrev, S. Havlin and H.E. Stanley1992

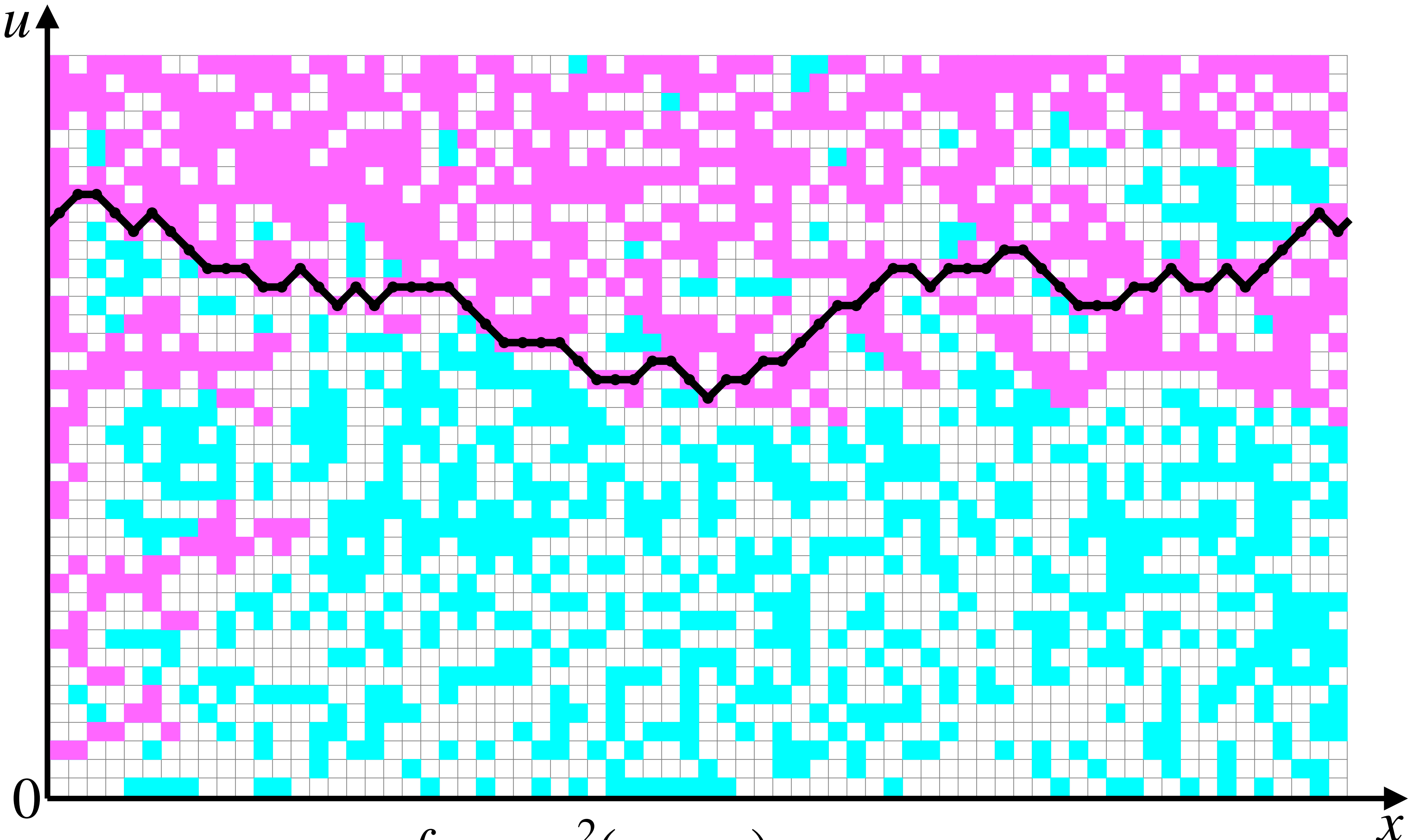
Anharmonic depinning = TL92

$$\mathscr{E}_{\rm el} = \sum_i \frac{c_4}{4}(u_i - u_{i+1})^4$$



anharmonic depinning respects the Middleton theorem
= return point memory (not guaranteed for qKPZ)

TL92 and directed percolation $(d = 1)$

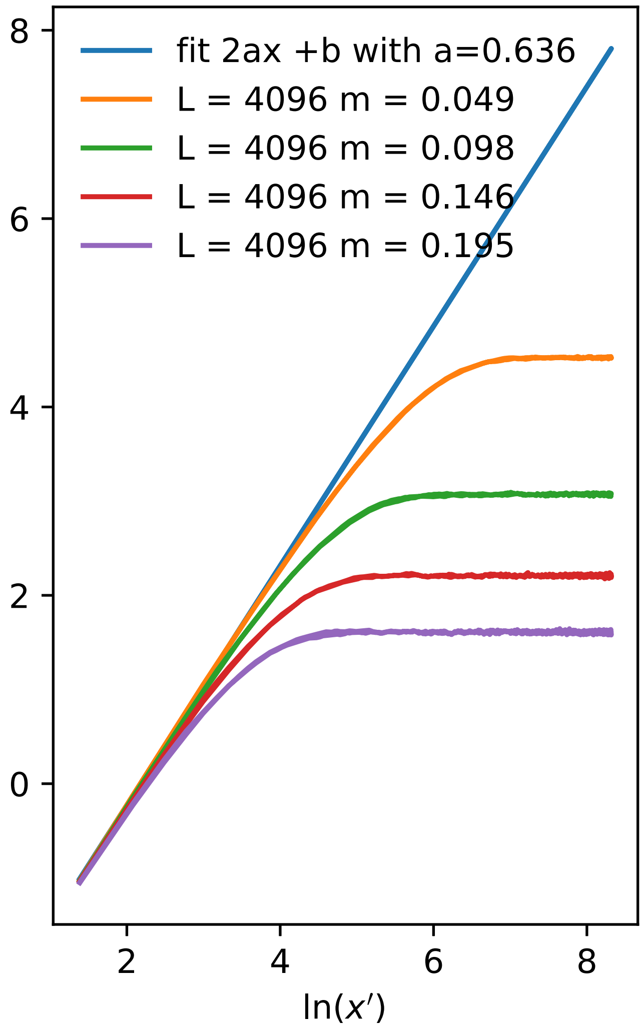


$$f_c := m^2(u - w)$$

used for driving

2-point function

$$\frac{1}{2}\overline{[u(x)-u(y)]^2} \sim \begin{cases} A\,|x-y|^{2\zeta}, & |x-y|<\xi \\ B\,m^{-2\zeta_m}, & |x-y|>\xi \end{cases}$$



from directed percolation

$$\zeta^{d=1} = \frac{\nu_\perp}{\nu_\parallel} = 0.632613(3)$$

$$\zeta_m^{d=1} = \frac{2\nu_\perp}{1+\nu_\perp} = 1.046190(4)$$

two distinct exponents in all d

$$\zeta_m > \zeta$$

What is the appropriate long-distance theory?

Can we measure it?

standard elasticity $c \to 0$

non-linear elasticity

$$\eta \partial_t u(x,t) = c \nabla^2 u(x,t) + c_4 \nabla [\nabla u(x,t)]^3 - m^2 [u(x,t) - w] + F\big(x, u(x,t)\big)$$

disorder force

confining potential

background field

What is the appropriate long-distance theory?

Can we measure it?

standard elasticity
$c > 0$

non-linear elasticity

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + c_4 \nabla[\nabla u(x,t)]^3 - m^2[u(x,t) - w(x,t)]$$
$$+\lambda[\nabla u(x,t)]^2 + F\big(x, u(x,t)\big)$$

KPZ term

disorder force

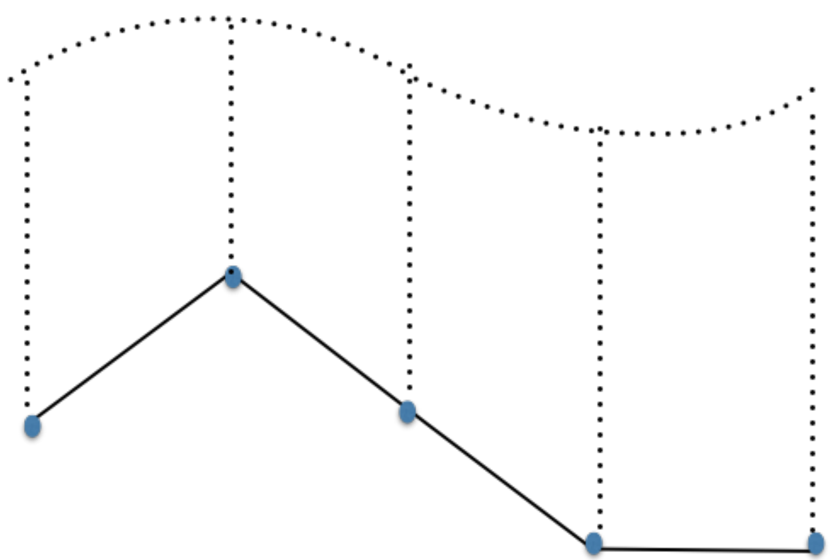
confining potential
(unrenormalized)

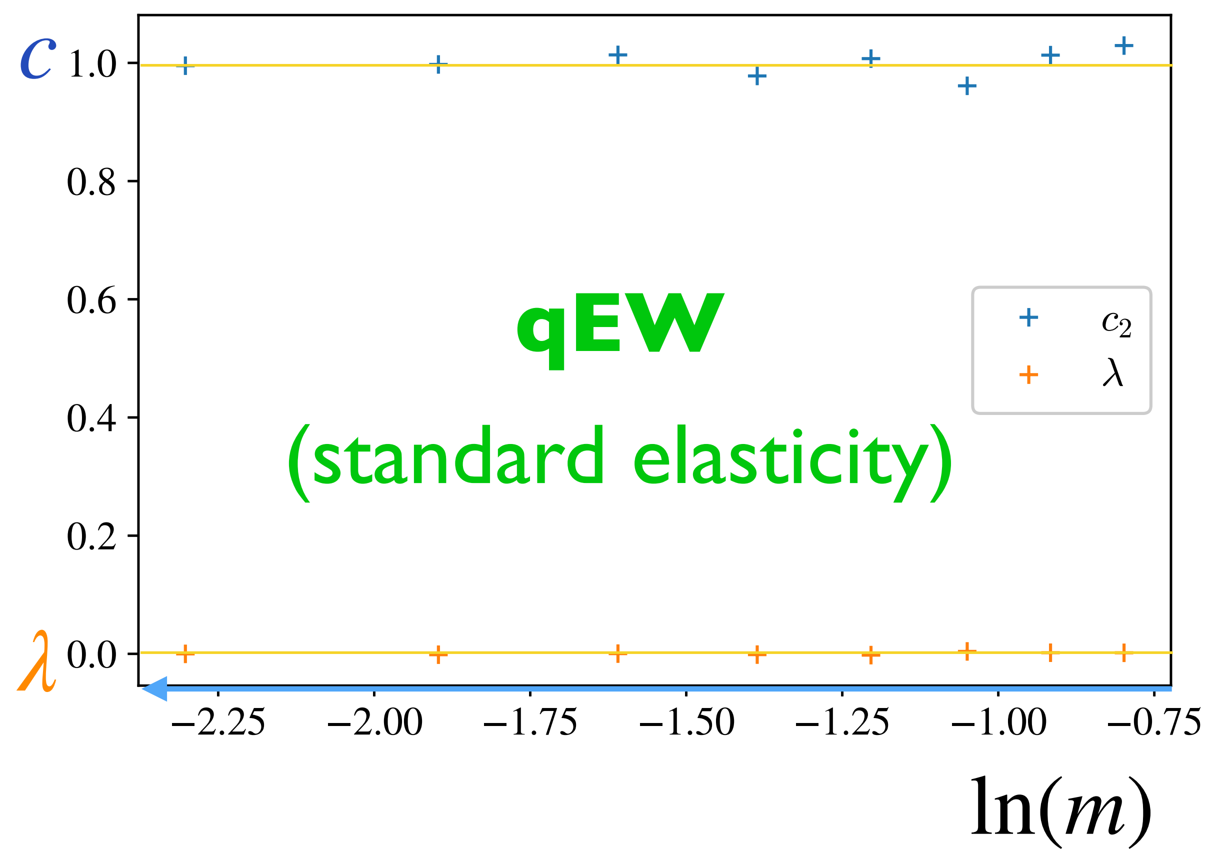
background field
(modulated)

Measuring the elastic constants for harmonic depinning (qEW)

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) - m^2[u(x,t) - w(x,t)] + F(x, u(x,t))$$

$$w(x) = w_0 + A sin(\frac{\pi x}{L})$$

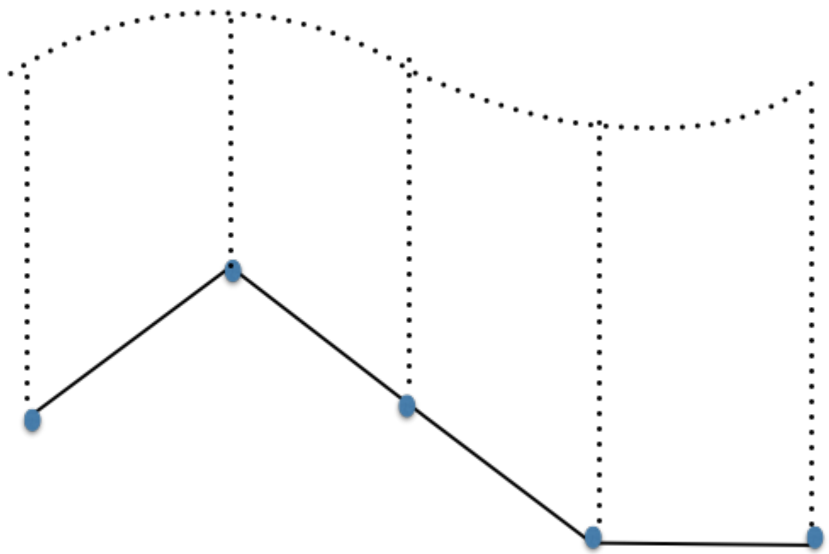


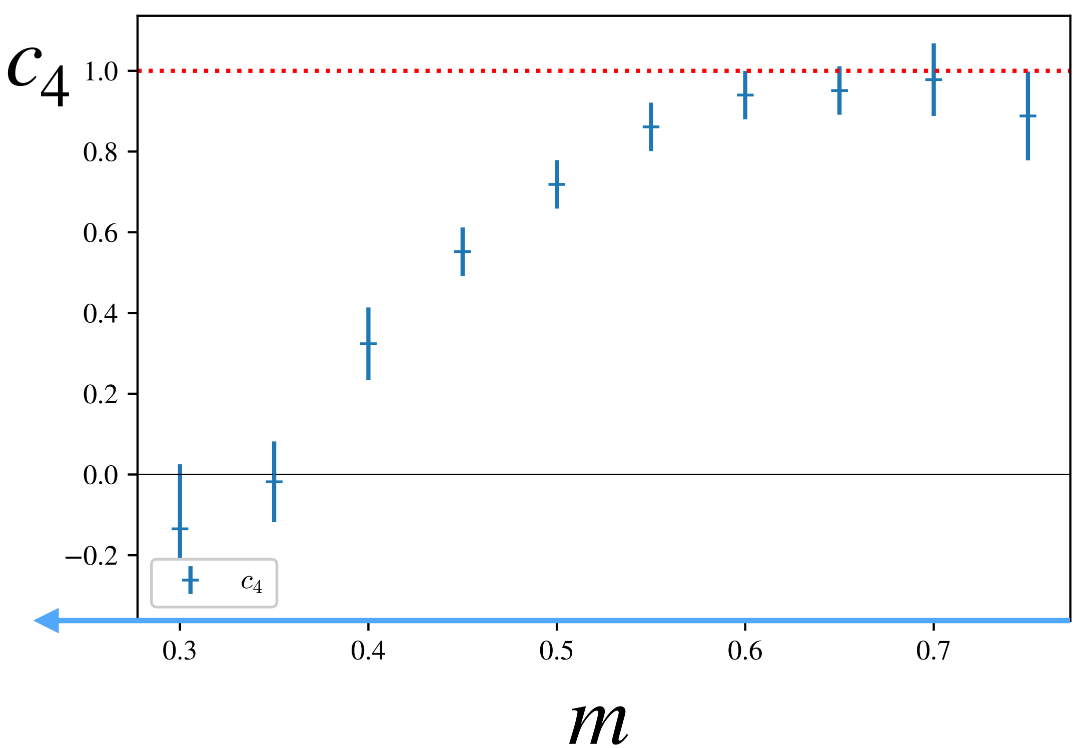


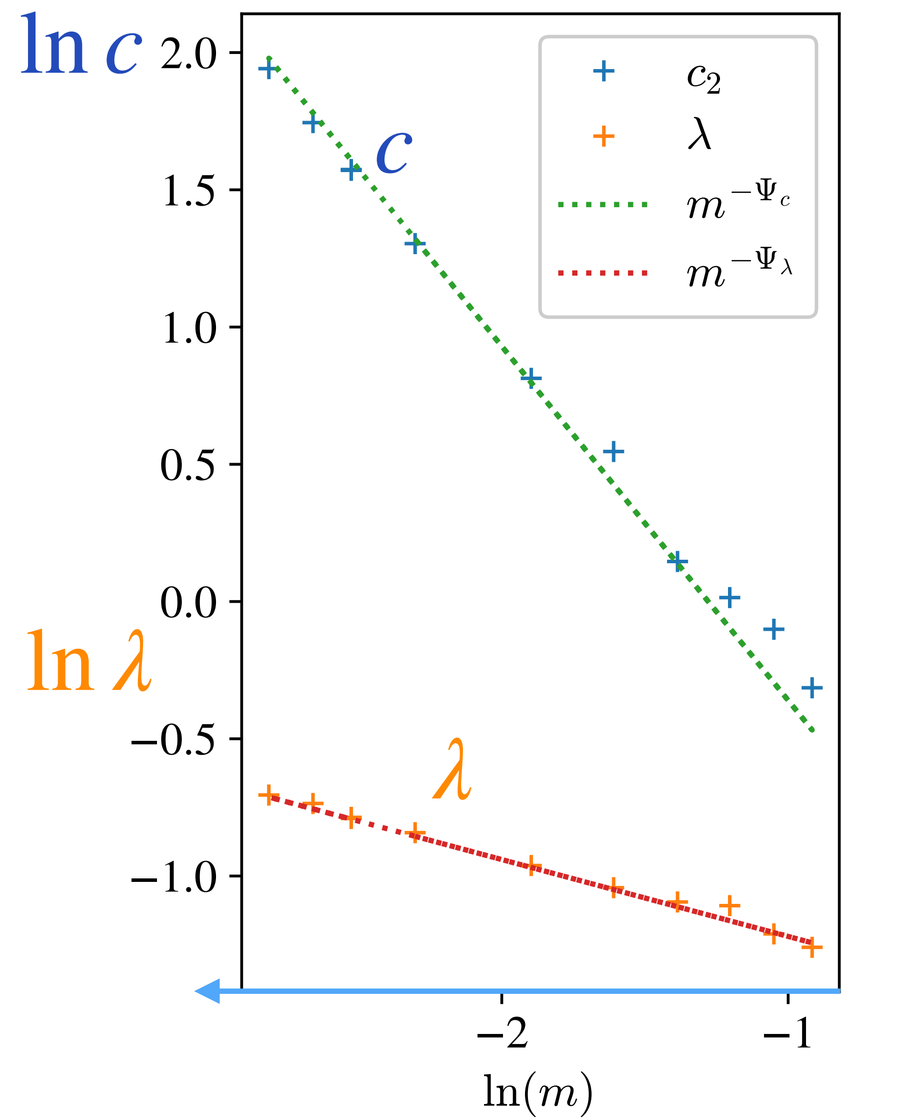
$$w(x) = w_0 + A sin(\frac{\pi x}{L})$$

Measuring the elastic constants

anharmonic depinning ($c_4 > 0$)





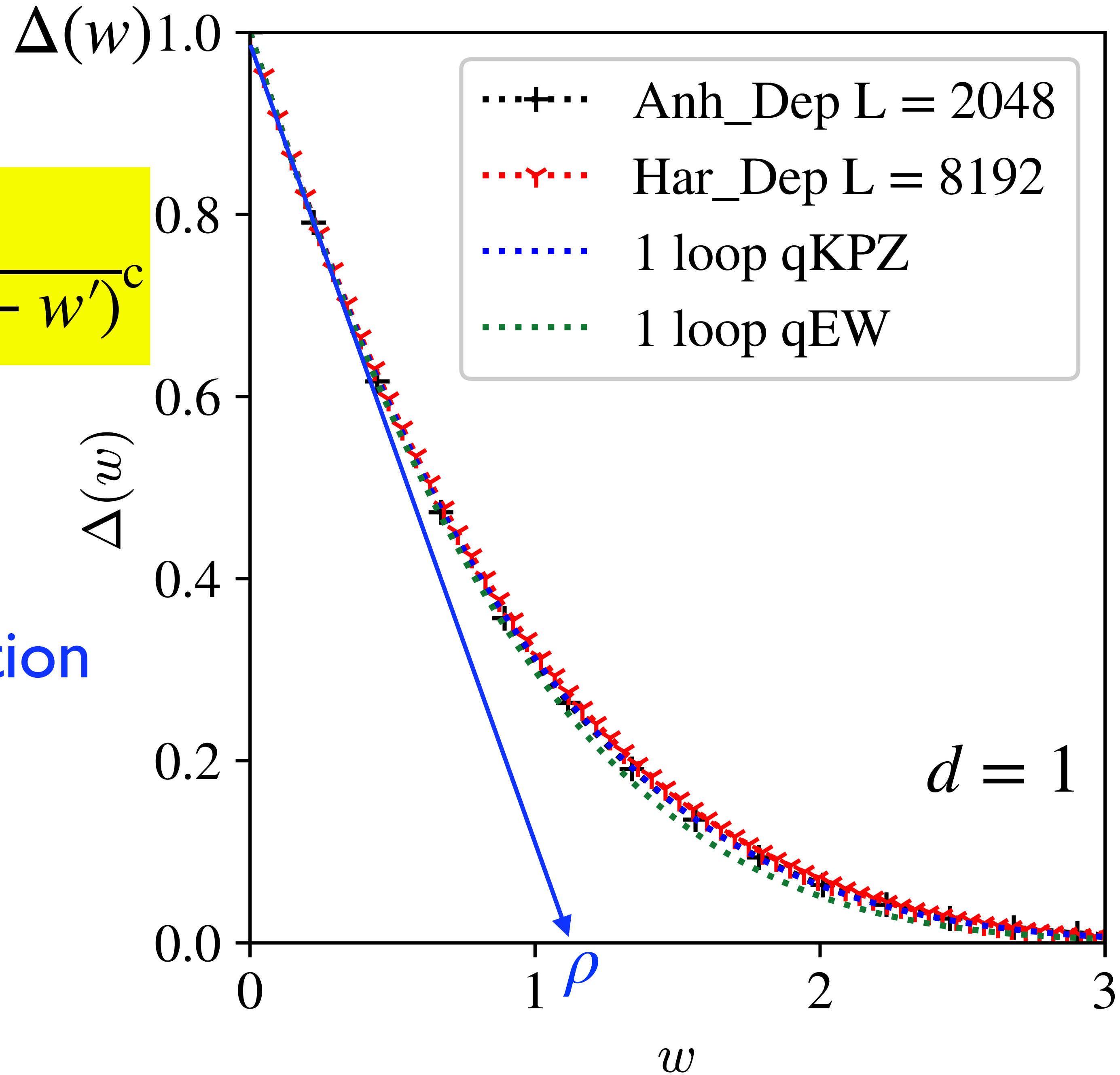


Measuring the effective force correlator

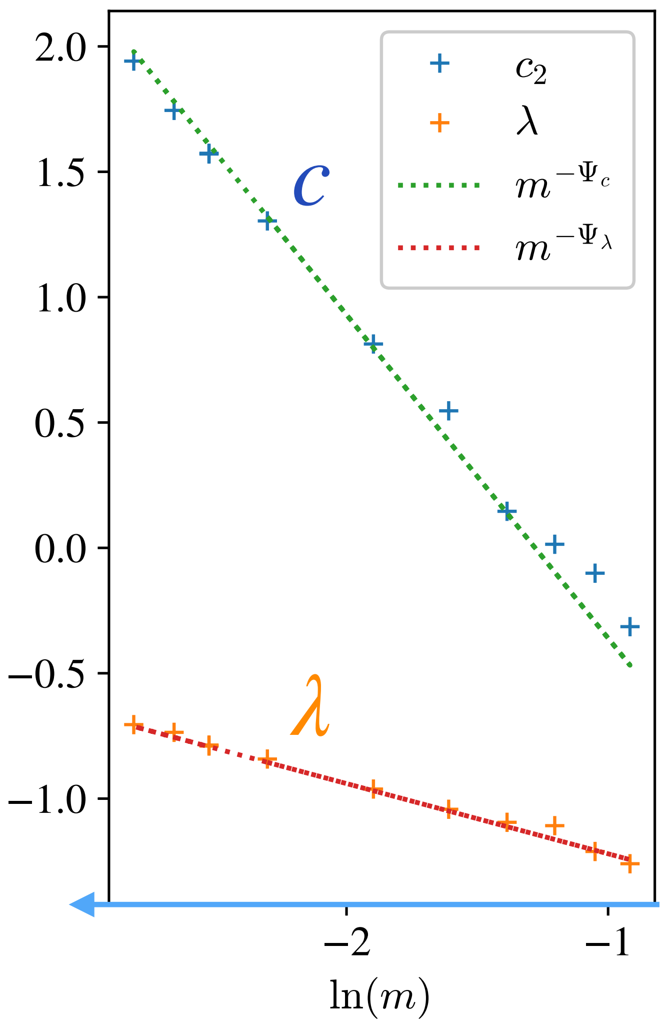
$$\Delta(w - w') = m^4 L^d \overline{(u_w - w)(u_{w'} - w')}^{\mathrm{c}}$$

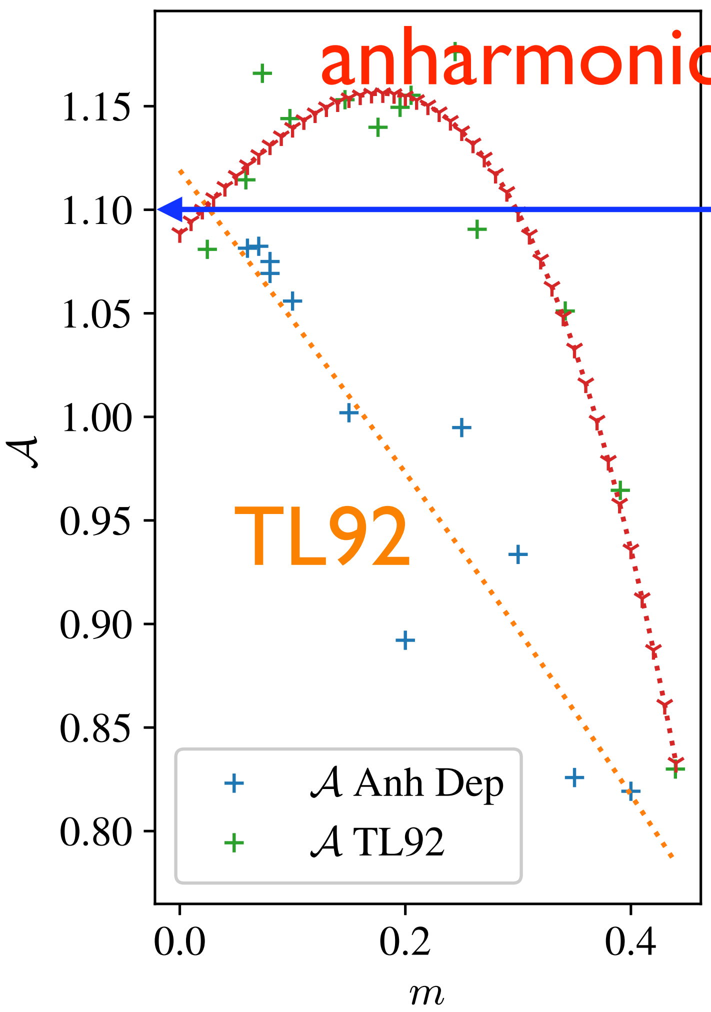
$$u_w = \frac{1}{L^d} \int_x u_w(x)$$

centre-of-mass position given w



Coupling constant for qKPZ





$$\mathscr{A}_{d=1} = 1.10(2)$$

scale-free universal KPZ amplitude

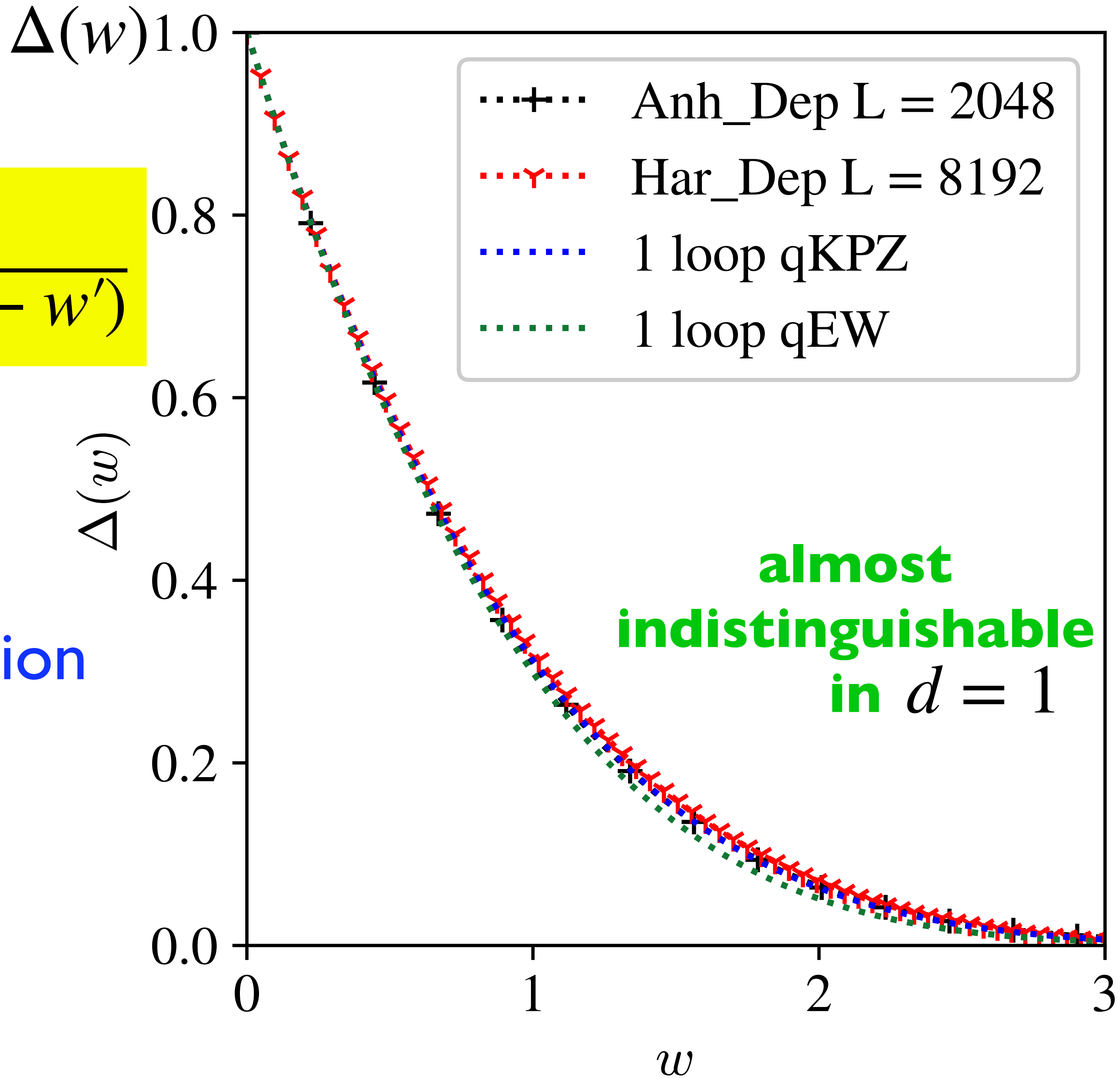
$$\mathscr{A} := \rho \frac{\lambda}{c} \equiv \frac{\Delta(0)}{|\Delta'(0^+)|} \frac{\lambda}{c}$$

Measuring the effective force correlator

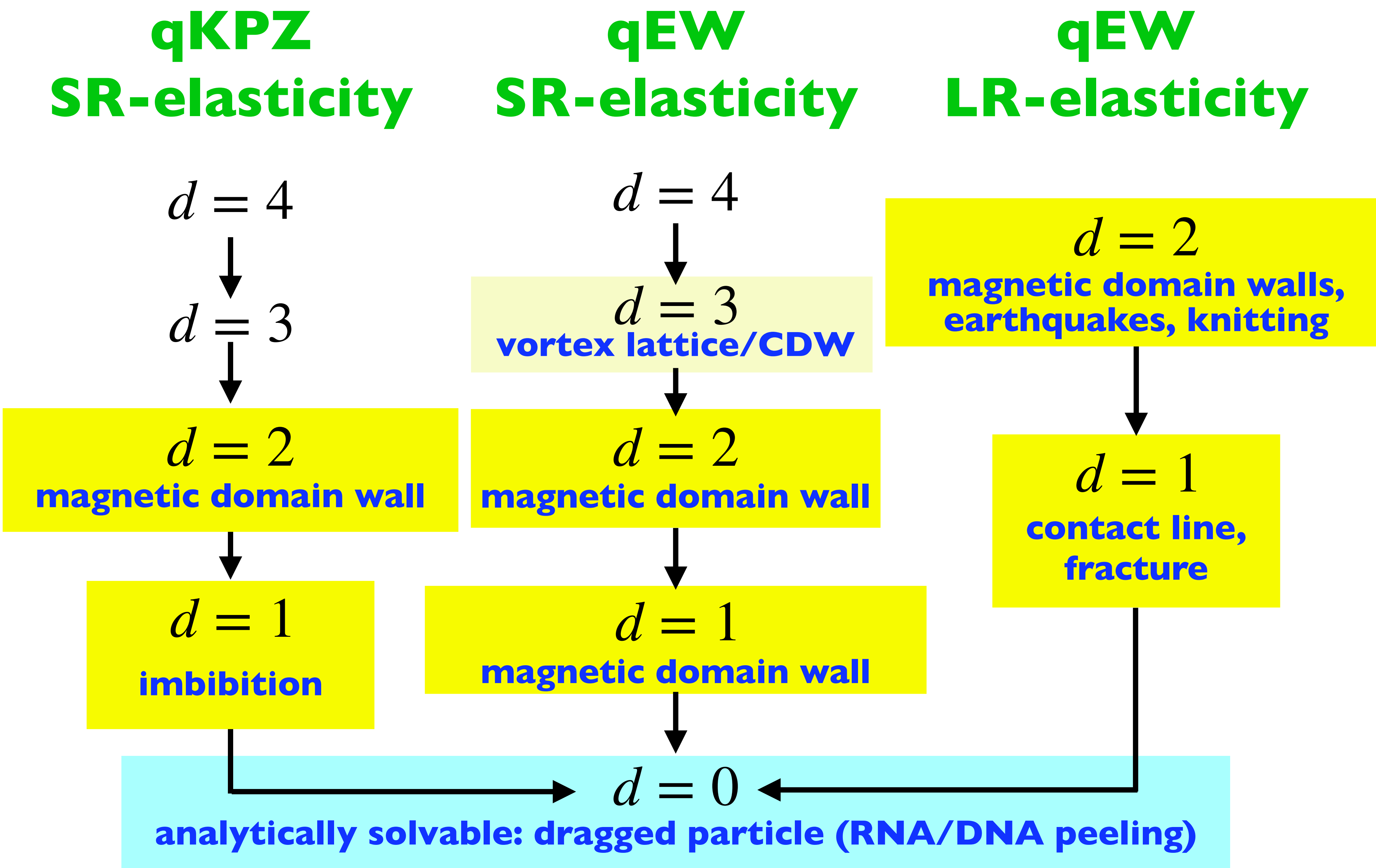
$$\Delta(w - w') = m^4 L^d \overline{(u_w - w)(u_{w'} - w')}$$

$$u_w = \frac{1}{L^d} \int_x u_w(x)$$

centre-of-mass position given w



Universality classes for depinning



FRG flow equations

shooting parameter

Flow of the disorder for qKPZ

$$\partial_\ell \tilde{\Delta}(u) = \left(4 - d\frac{\zeta_m}{\zeta} - 2\zeta_m\right)\tilde{\Delta}(u) + u\zeta_m \tilde{\Delta}'(u)$$
$$+\frac{d(d+2)}{12}\tilde{\lambda}^2\tilde{\Delta}(u)^2 - \tilde{\Delta}'(u)^2 - \tilde{\Delta}''(u)\left[\tilde{\Delta}(u) - \tilde{\Delta}(0)\right]$$

replace ζ_m/ζ

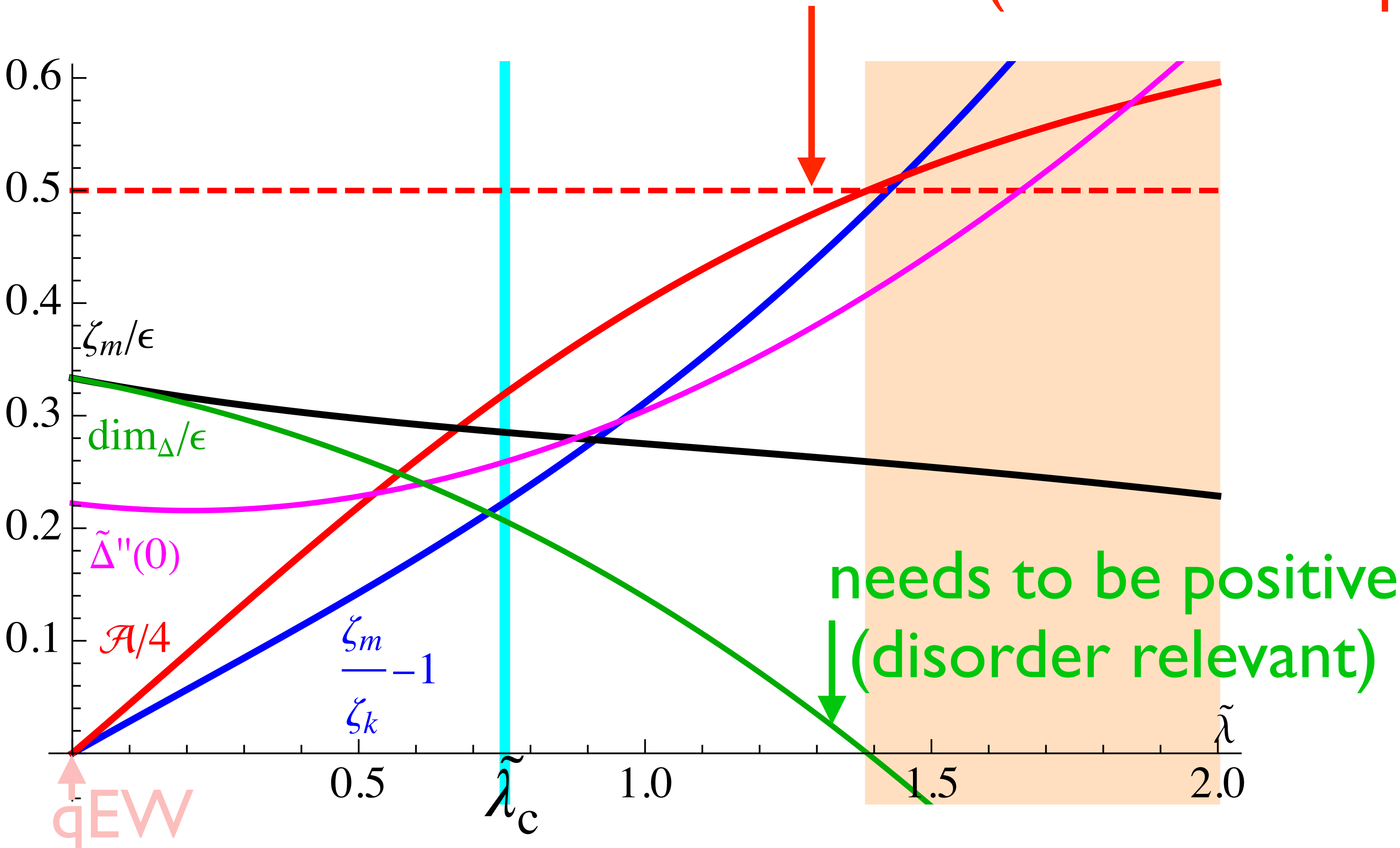
$$\frac{\zeta_m}{\zeta} = 1 + \frac{1}{2}\left[-\tilde{\lambda}\tilde{\Delta}'\left(0^+\right) - \frac{d-1}{3}\tilde{\lambda}^2\tilde{\Delta}(0)\right].$$

flow for λ (with confining potential, i.e. massive theory)

$$-m\partial_m \tilde{\lambda} = \zeta_m \tilde{\lambda} - \frac{4-d}{6}\tilde{\lambda}^3\tilde{\Delta}(0) \implies \tilde{\lambda}_c = \sqrt{\frac{6\zeta_m}{(4-d)\tilde{\Delta}(0)}}$$

Solution in $d = 1$

$\mathscr{A} < 2$ (critical force positive)



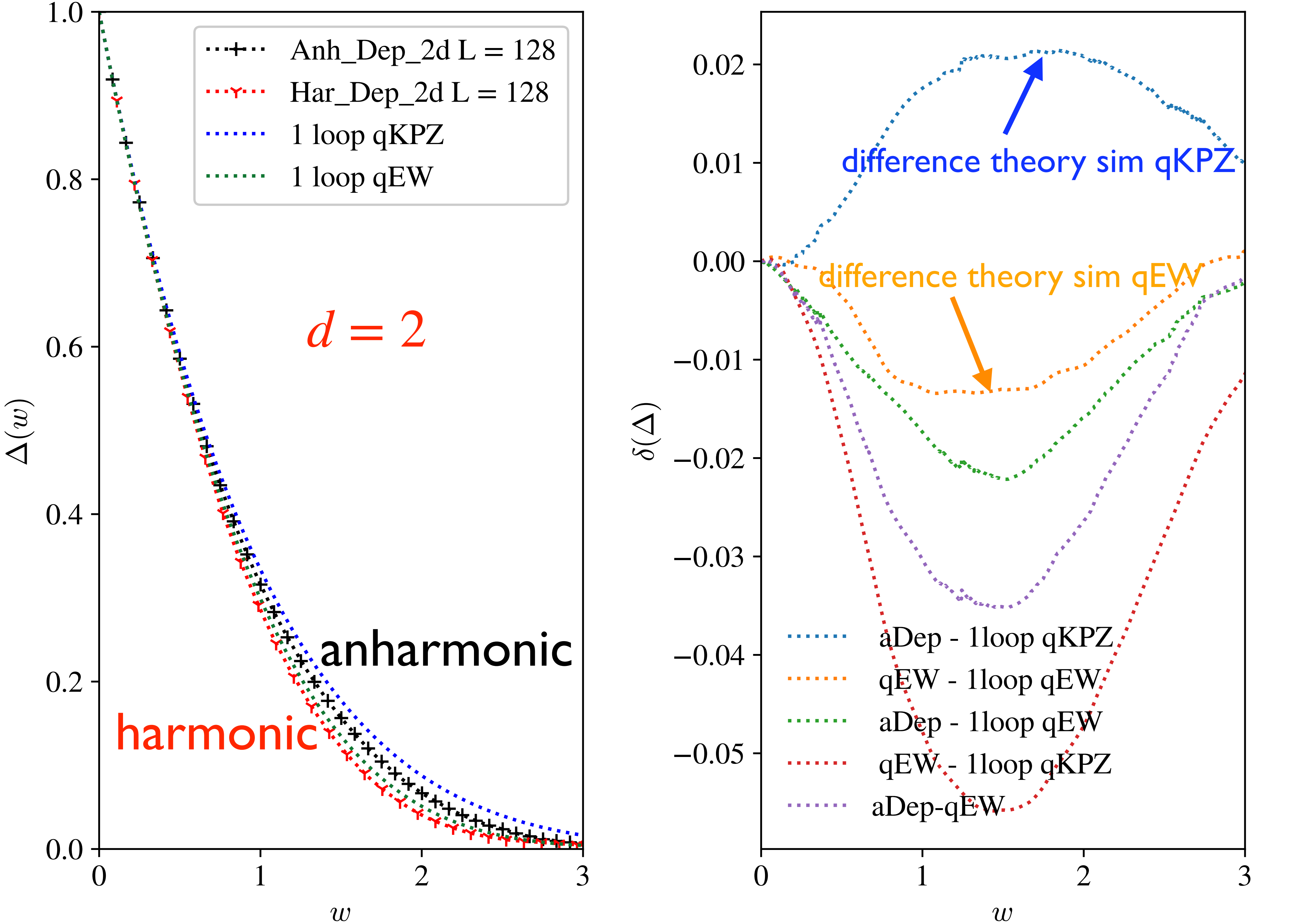
RG:

$$\zeta_m^{d=1} = 0.86$$
$$\zeta^{d=1} = 0.69$$
$$z^{d=1} = 1.27$$
$$\mathscr{A}^{d=1} = 1.27$$

numerics:

$$\zeta_m^{d=1} = 1.05$$
$$\zeta^{d=1} = 0.63$$
$$z^{d=1} = 1.10(2)$$
$$\mathscr{A}^{d=1} = 1.10(2)$$

Shape of $\Delta(w)$ different in $d=2$



Conclusions

- when in doubt: measure effective long-distance action (= theory/description)
- standard elastic depinning (**qEW**) has non-trivial disorder correlator given by FRG
- imbibition (e.g. TL92), anharmonic depinning and qKPZ all belong to the same universality class: the effective long-wavelength theory is **qKPZ**
- you need to introduce a confining potential $m^2[w - u(x,t)]$ to measure disorder correlations
  $\Rightarrow$ give up the Cole-Hopf transform
  $\Rightarrow$ yields an RG fixed point
- a field theory can be build

Theory and Experiments for Disordered Elastic Manifolds, Depinning, Avalanches, and Sandpiles

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1 September 2021 – masterENS.tex – REVISION 1.1083

Abstract. Domain walls in magnets, vortex lattices in superconductors, contact lines at depinning, and many other systems can be modeled as an elastic system subject to quenched disorder. The ensuing field theory posesses a well-controlled perturbative expansion around its upper critical dimension. Contrary to standard field theory, the renormalization group flow involves a function, the disorder correlator $\Delta(w)$, and is therefore termed the functional renormalization group (FRG). $\Delta(w)$ is a physical observable, the auto-correlation function of the center of mass of the elastic manifold. In this review, we give a pedagogical introduction into its phenomenology and techniques. This allows us to treat both equilibrium (statics), and depinning (dynamics). Building on these techniques, avalanche observables are accessible: distributions of size, duration, and velocity, as well as the spatial and temporal shape. Various equivalences between disordered elastic manifolds, and sandpile models exist: an elastic string driven at a point and the Oslo model; disordered elastic manifolds and Manna sandpiles; charge density waves and Abelian sandpiles or loop-erased random walks. Each of the mappings between these systems requires specific techniques, which we develop, including modeling of discrete stochastic systems via coarse-grained stochastic equations of motion, super-symmetry techniques, and cellular automata. Stronger than quadratic nearest-neighbor interactions lead to directed percolation, and non-linear surface growth with additional KPZ terms. On the other hand, KPZ without disorder can be mapped back to disordered elastic manifolds, either on the directed polymer for its steady state, or a single particle for its decay. Other topics covered are the relation between functional RG and replica symmetry breaking, and random field magnets. Emphasis is given to numerical and experimental tests of the theory.

Review (133 pages)

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latest version:

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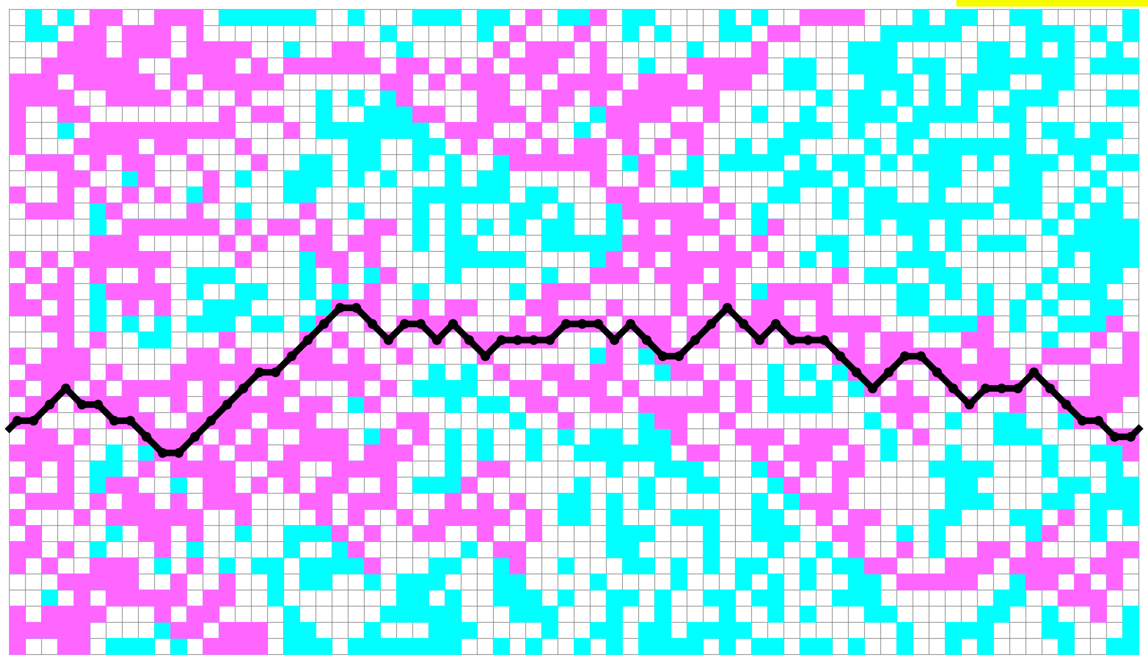

Anisotropic depinning with its relation to directed percolation, explained in section 5.7.

pedagogic introduction in basic sections!

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