Interaction, disorder, elasticity GDR meeting
28/11/2022

Hidden non-equilibrium phase transition to temporal oscillations in a disordered mean-field spin model

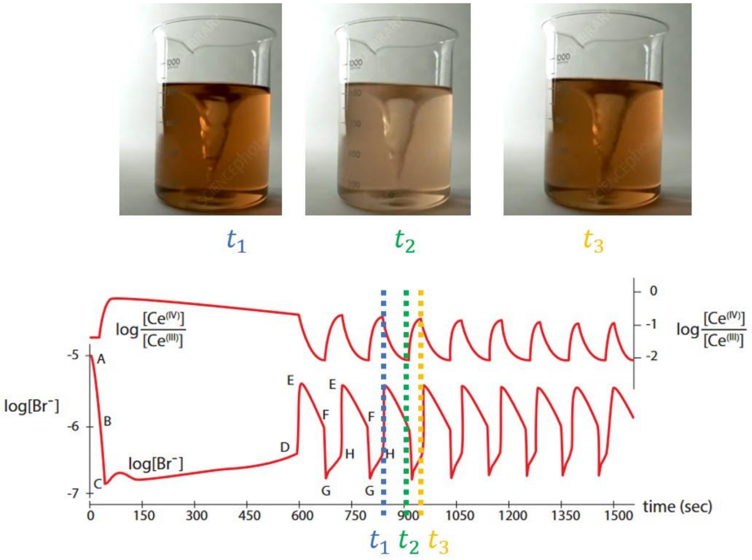
Ph.D student Laura Guislain

SUPERVISED BY ERIC BERTIN

Laboratoire interdisciplinaire de Physique

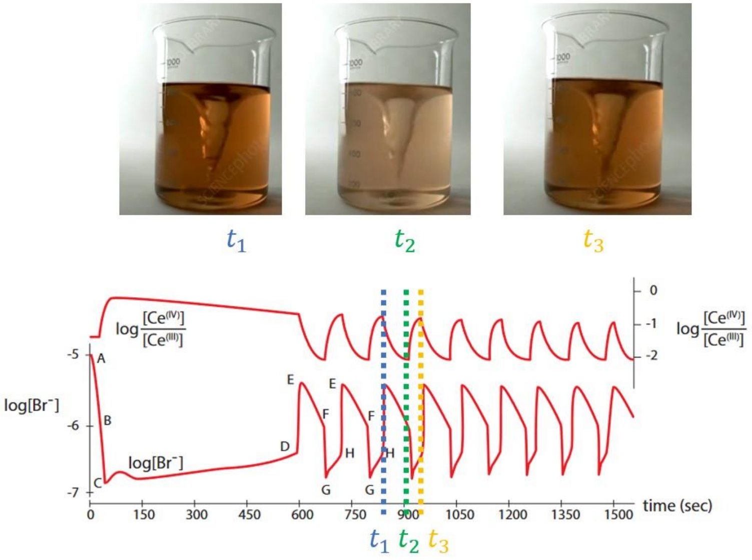
Université Grenoble Alpes

Hidden non-equilibrium phase transition to temporal oscillations in a disordered mean-field spin model



Belousov-Zhabotinsky[1] chemical reaction

Hidden non-equilibrium phase transition to temporal oscillations in a disordered mean-field spin model



Belousov-Zhabotinsky[1] chemical reaction

It is a classical example of:

- an out-of-equilibrium dissipative system
- with a large assembly of interacting units
- which exhibits spontaneous oscillations at a collective scale.

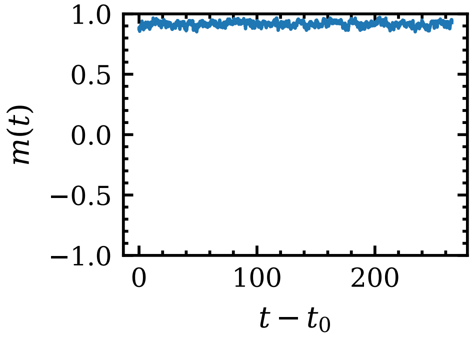
Hidden non-equilibrium phase transition to temporal oscillations in a disordered mean-field spin model

Example of introducing disorder at equilibrium:

<u>Equilibrium model:</u>

Ising model

$$\mathcal{H} = -\sum_{i,j} J\, s_i s_j$$



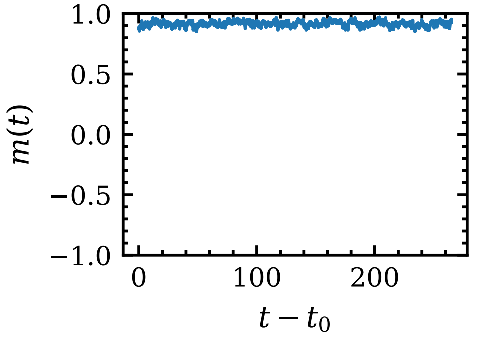
The Ising model exhibits a paramagnetic $(m = 0)$ to ferromagnetic $(m \neq 0)$ phase transition.

Hidden non-equilibrium phase transition to temporal oscillations in a disordered mean-field spin model

Example of introducing disorder at equilibrium:

Equilibrium model:
Ising model

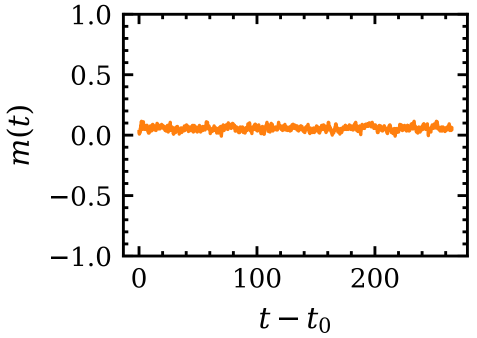
$$\mathcal{H} = -\sum_{i,j} J\, s_i s_j$$



The Ising model exhibits a paramagnetic ($m = 0$) to ferromagnetic ($m \neq 0$) phase transition.

Disordered model:
Quenched disordered model

$$\mathcal{H} = -\sum_{i,j} J(i,j)\, s_i s_j$$



Effects of the disorder:

- the ferromagnetic order disappears
- other parameters carry the signature of a phase transition (like the overlap distribution, or the Edward-Anderson parameter $q_{EA} = \overline{\langle s_i \rangle^2}$).

There is a hidden phase transition.

Questions:

What is the effect of disorder in out-of-equilibrium systems exhibiting collective oscillations ?

- Does the oscillating order disappear ?
- Are there other quantities still carrying the signature of the phase transition ?

We investigate these general questions in a specific disordered non-equilibrium mean field spin model exhibiting oscillations.

Questions:

What is the effect of disorder in out-of-equilibrium systems exhibiting collective oscillations ?

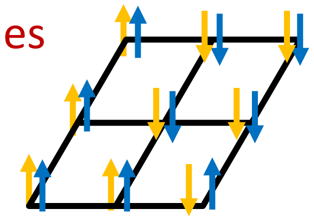
- Does the oscillating order disappear ?
- Are there other quantities still carrying the signature of the phase transition ?

We investigate these general questions in a specific disordered non-equilibrium mean field spin model exhibiting oscillations.

Non-equilibrium mean-field spin model exhibiting oscillations

We consider a simple model, studied in [1]:

2N variables



N spins $s_i = \pm 1$
N fields $h_i = \pm 1$

Interaction energy

$$NE_s = -\underbrace{J_1 \sum_{i,j} s_i s_j}_{\text{spin-spin } \uparrow\uparrow\ J_1>0} - \underbrace{\sum_{i,j} s_i h_j}_{\text{spin-field } \uparrow\uparrow 1>0}$$

$$NE_h = -\underbrace{J_2 \sum_{i,j} h_i h_j}_{\text{field-field } \uparrow\uparrow J_2>0} - \underbrace{(1-\mu) \sum_{i,j} s_i h_j}_{\text{field-spin } \uparrow\downarrow\ 1-\mu<0}$$

→ this frustration can give oscillations

Control parameters:

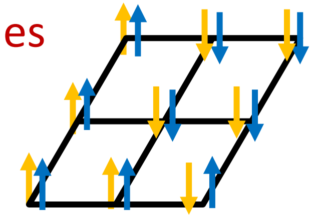
- T the temperature
- μ the distance to equilibrium

[1] L. Guislain & E. Bertin 2022, *submitted*

Non-equilibrium mean-field spin model exhibiting oscillations

We consider a simple model, studied in [1]:

2N variables



N spins $s_i = \pm 1$

N fields $h_i = \pm 1$

Interaction energy

$$NE_s = -\underbrace{J_1 \sum_{i,j} s_i s_j}_{\text{spin–spin } \uparrow\uparrow\ J_1>0} - \underbrace{\sum_{i,j} s_i h_j}_{\text{spin–field } \uparrow\uparrow 1>0}$$

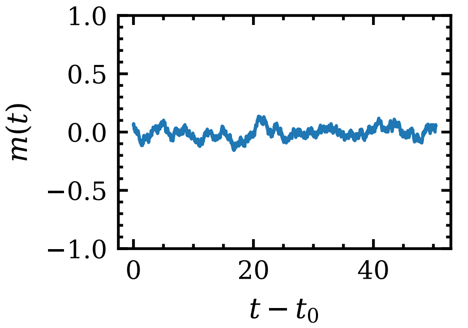
$$NE_h = -\underbrace{J_2 \sum_{i,j} h_i h_j}_{\text{field–field } \uparrow\uparrow J_2>0} - \underbrace{(1-\mu) \sum_{i,j} s_i h_j}_{\text{field–spin } \uparrow\downarrow\ 1-\mu<0}$$

→ this frustration can give oscillations

Control parameters:

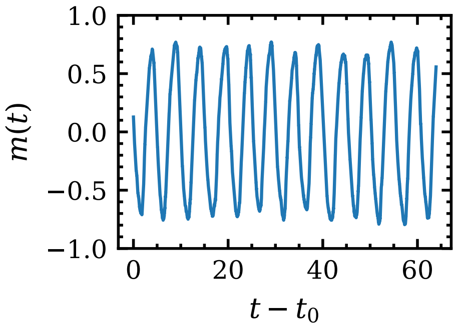
- T the temperature
- μ the distance to equilibrium

Behavior: for μ high enough, there is a phase transition from a paramagnetic to an oscillating phase at $T_c = 1 + J_1 + J_2$.



$T > T_c$

$\langle m^2 \rangle = O(N^{-1})$



$T < T_c$

$\langle m^2 \rangle = O(N^0)$

[1] L. Guislain & E. Bertin 2022, *submitted*

A simple model with disorder

Recap: model without disorder

$$E_s = -\frac{1}{N}\left(J_1 \sum_{i,j} s_i s_j + \sum_{i,j} s_i h_j\right)$$

$$E_h = -\frac{1}{N}\left(J_2 \sum_{i,j} h_i h_j + (1-\mu) \sum_{i,j} s_i h_j\right)$$

The mean-field model exhibits a phase transition to an oscillating state.

A simple model with disorder

Recap: model without disorder

$$E_s = -\frac{1}{N}\left(J_1 \sum_{i,j} s_i s_j + \sum_{i,j} s_i h_j\right)$$

$$E_h = -\frac{1}{N}\left(J_2 \sum_{i,j} h_i h_j + (1-\mu)\sum_{i,j} s_i h_j\right)$$

The mean-field model exhibits a phase transition to an oscillating state.

Modified model: simple model with separable quenched disorder[1]

$$E_s = -\frac{1}{N}\left(J_1 \sum_{i,j} \epsilon_i \epsilon_j s_i s_j + \sum_{i,j} \epsilon_i \epsilon_j s_i h_j\right)$$

$$E_h = -\frac{1}{N}\left(J_2 \sum_{i,j} \epsilon_i \epsilon_j h_i h_j + (1-\mu)\sum_{i,j} \epsilon_i \epsilon_j s_i h_j\right)$$

New random variable on each site $\epsilon_i = \pm 1$ with $\overline{\epsilon_i} = 0$

[1]Inspired from 'Solvable spin systems with random interactions '' D.C Mattis 1976

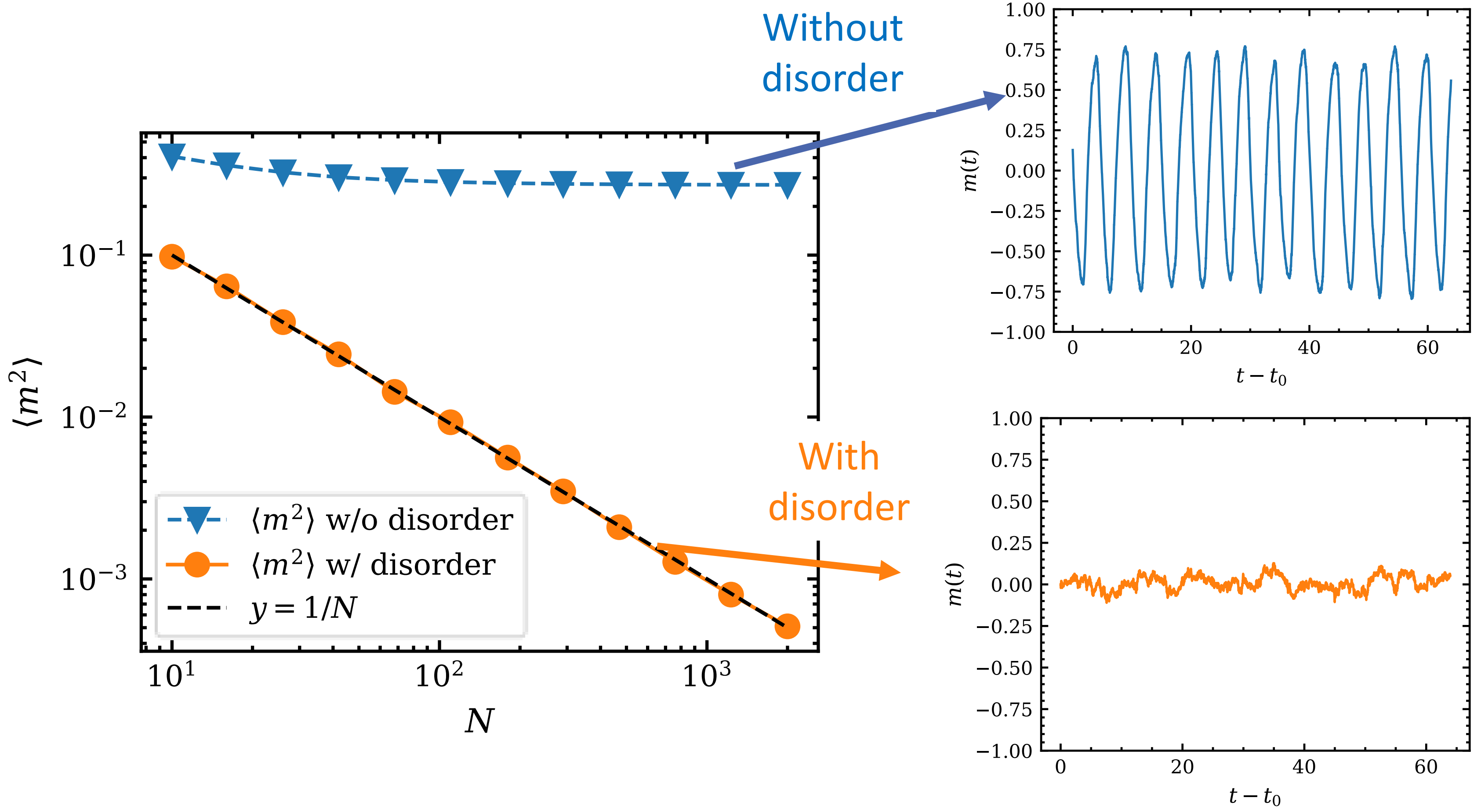
Questions:

What is the effect of disorder in out-of-equilibrium systems exhibiting collective oscillations ?

- Does the oscillating order disappear ?
- Are there other quantities still carrying the signature of the phase transition ?

Does the oscillating order disappear ?

We compute $m(t)$ and $\langle m^2 \rangle$ using Monte-Carlo simulations for $\mu = 1.4$ and $T = 0.3 < T_c$.



Does the oscillating order disappear ?

Yes:

If we only consider the order parameter $\langle m^2 \rangle$, it looks as if there is no transition.

Questions:

What is the effect of disorder in out-of-equilibrium systems exhibiting collective oscillations ?

- Does the oscillating order disappear ?
- Are there other quantities still carrying the signature of the phase transition ?

Are there other quantities still carrying the signature of the phase transition ?

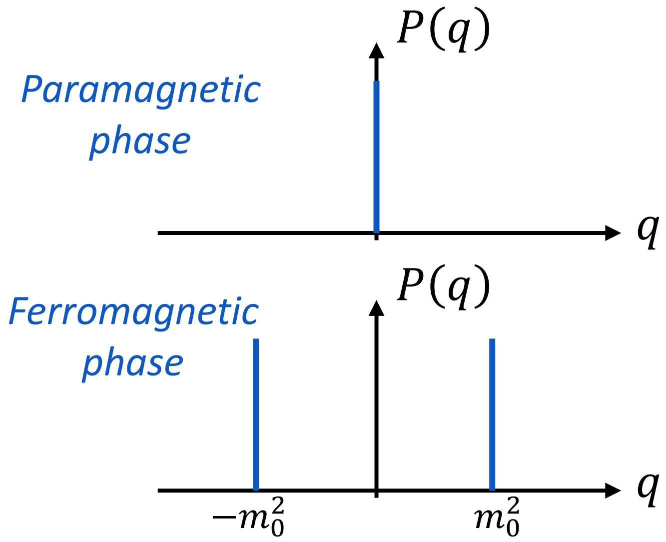
We consider the overlap distribution which measures the correlation between two configurations a and b:

$$P(q) = \sum_{\{s_i^a\},\{s_i^b\}} P(\{s_i^a\})P(\{s_i^b\})\delta\left(\underbrace{\frac{1}{N}\sum_i s_i^a s_i^b}_{q_{ab}} - q\right)$$

q_{ab} is the overlap between two spin configurations $\{s_i^a\}$ and $\{s_i^b\}$:

- Identical configurations $q_{ab} = 1$
- Uncorrelated configurations $q_{ab} = 0$

Examples of $P(q)$



Are there other quantities still carrying the signature of the phase transition ?

We consider the overlap distribution which measures the correlation between two configurations a and b:

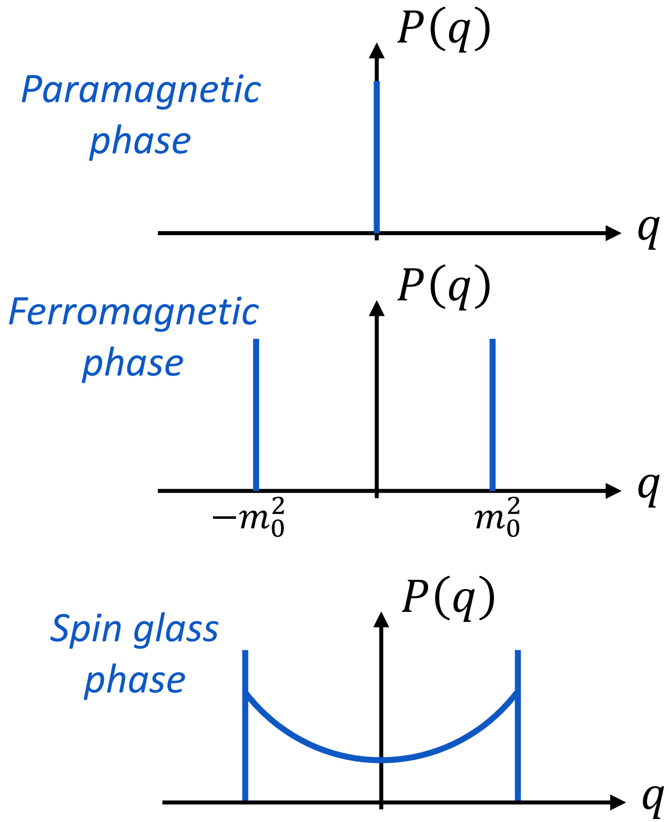
$$P(q) = \sum_{\{s_i^a\},\{s_i^b\}} P(\{s_i^a\})P(\{s_i^b\})\delta\left(\underbrace{\frac{1}{N}\sum_i s_i^a s_i^b}_{q_{ab}} - q\right)$$

q_{ab} is the overlap between two spin configurations $\{s_i^a\}$ and $\{s_i^b\}$:

- Identical configurations $q_{ab} = 1$
- Uncorrelated configurations $q_{ab} = 0$

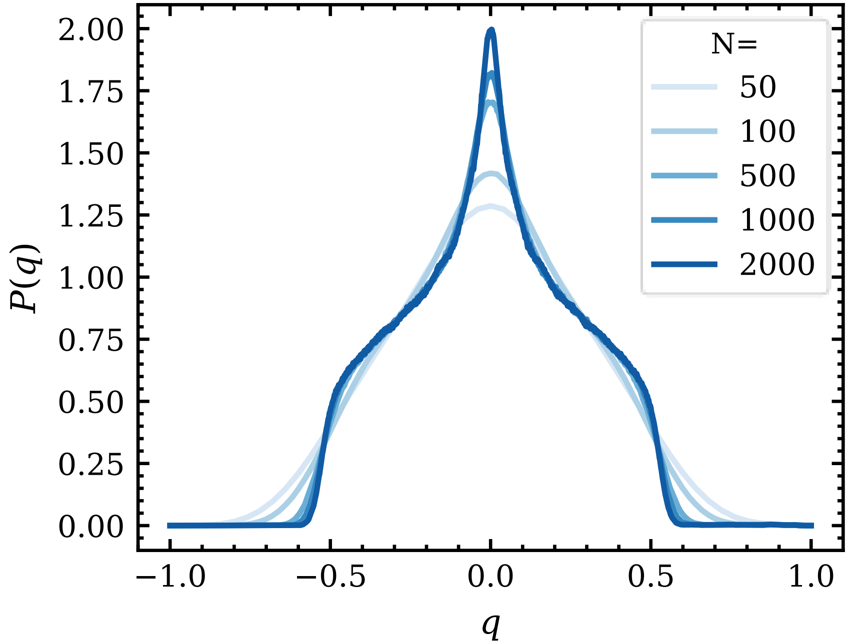
The nontrivial nature of the overlap structure reflects the presence of many states that are not related to each other by a simple symmetry transformation.

Examples of $P(q)$



Are there other quantities still carrying the signature of the phase transition ?

We compute $P(q)$ using Monte-Carlo simulations for $\mu = 1.4$ and $T = 0.3 < T_c$.

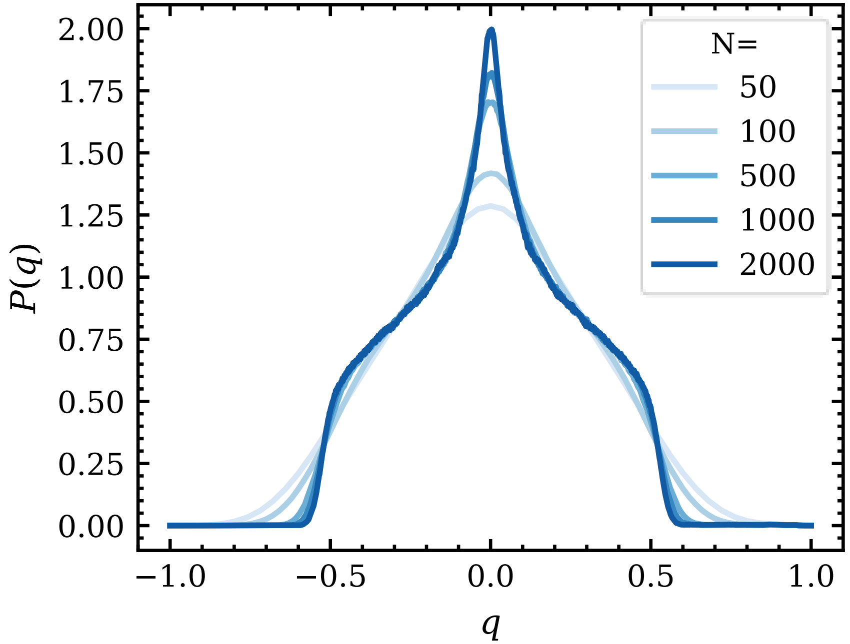


Observation:

There is a non trivial structure:
the probability density is spread over an interval

Are there other quantities still carrying the signature of the phase transition ?

We compute $P(q)$ using Monte-Carlo simulations for $\mu = 1.4$ and $T = 0.3 < T_c$.



Observation:

There is a non trivial structure: the probability density is spread over an interval

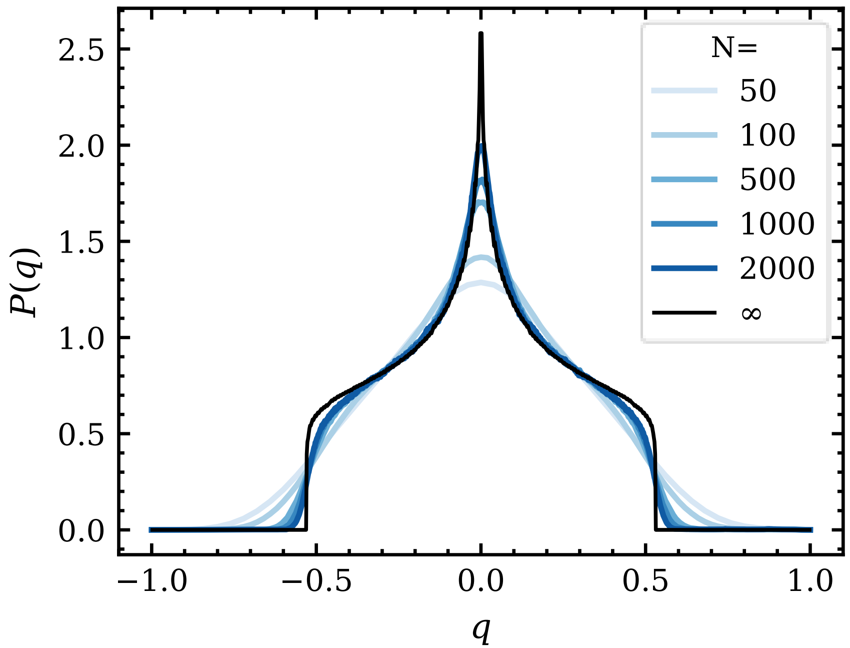
This raises the following questions:

How to explain this non-trivial distribution of the overlap ?

- Is it due to the presence of disorder like in spin-glasses ?
- Or is it due to the presence of oscillations ?

Are there other quantities still carrying the signature of the phase transition ?

We compute $P(q)$ using Monte-Carlo simulations for $\mu = 1.4$ and $T = 0.3 < T_c$.



<u>Observation</u>:
There is a non trivial structure:
the probability density is spread over an interval

This raises the following questions:
How to explain this non-trivial distribution of the overlap ?

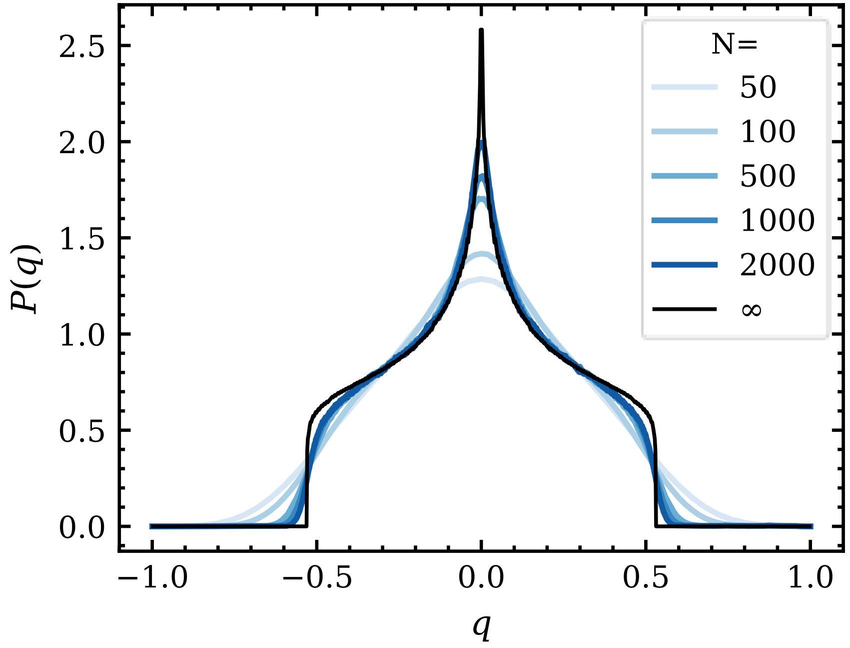
- Is it due to the presence of disorder like in spin-glasses ?
- Or is it due to the presence of oscillations ?

We analytically show that:

- The overlap distribution is the same with or without disorder
- In the limit $N \to \infty$, it gives a non-zero overlap distribution over an interval in an oscillating phase

Are there other quantities still carrying the signature of the phase transition ?

We compute $P(q)$ using Monte-Carlo simulations for $\mu = 1.4$ and $T = 0.3 < T_c$.



Observation:

There is a non trivial structure:
the probability density is spread over an interval

This raises the following questions:

How to explain this non-trivial distribution of the overlap ?

- Is it due to the presence of disorder like in spin-glasses ?
- Or is it due to the presence of oscillations ?

We analytically show that:

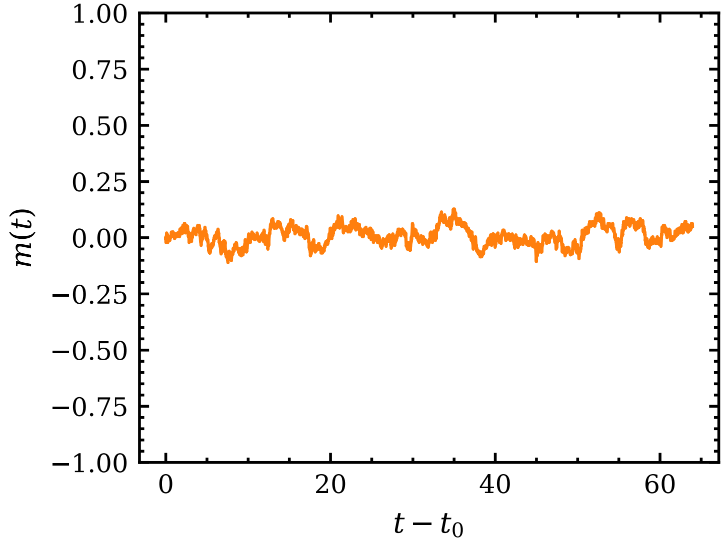
- The overlap distribution is the same with or without disorder
- In the limit $N \to \infty$, it gives a non-zero overlap distribution over an interval in an oscillating phase

Conclusion: the non-trivial overlap distribution is not due to the presence of disorder, but to the presence of **hidden temporal oscillations.**

It is possible to recover the oscillations ?

$$E_s = -\frac{1}{N}\left(J_1 \sum_{i,j} \epsilon_i s_i \epsilon_j s_j + \sum_{i,j} \epsilon_i s_i \epsilon_j h_j\right)$$

$$E_h = -\frac{1}{N}\left(J_2 \sum_{i,j} \epsilon_i h_i \epsilon_j h_j + (1-\mu) \sum_{i,j} \epsilon_i s_i \epsilon_j h_j\right)$$



$T < T_c$
and
$N = 470$

It is possible to recover the oscillations ?

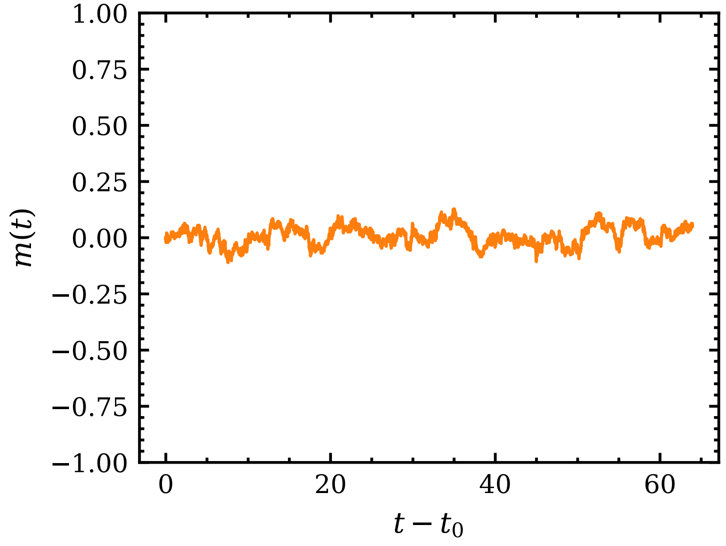
$$E_s = -\frac{1}{N}\left(J_1 \sum_{i,j} \overbrace{\epsilon_i s_i}^{\tilde{s}_i} \overbrace{\epsilon_j s_j}^{\tilde{s}_j} + \sum_{i,j} \overbrace{\epsilon_i s_i}^{\tilde{s}_i} \overbrace{\epsilon_j h_j}^{\tilde{h}_j}\right)$$

$$E_h = -\frac{1}{N}\left(J_2 \sum_{i,j} \underbrace{\epsilon_i h_i}_{\tilde{h}_i} \underbrace{\epsilon_j h_j}_{\tilde{h}_j} + (1-\mu) \sum_{i,j} \underbrace{\epsilon_i s_i}_{\tilde{s}_i} \underbrace{\epsilon_j h_j}_{\tilde{h}_j}\right)$$

We define new variables requiring the knowledge of the disorder

$\tilde{s}_i = \epsilon_i s_i$ and $\tilde{h}_i = \epsilon_i h_i$

$$\tilde{m} = N^{-1} \sum_i \epsilon_i s_i$$



$T < T_c$
and
$N = 470$

It is possible to recover the oscillations ?

$$E_s = -\frac{1}{N}\left(J_1 \sum_{i,j} \tilde{s}_i \tilde{s}_j + \sum_{i,j} \tilde{s}_i \tilde{h}_j\right)$$

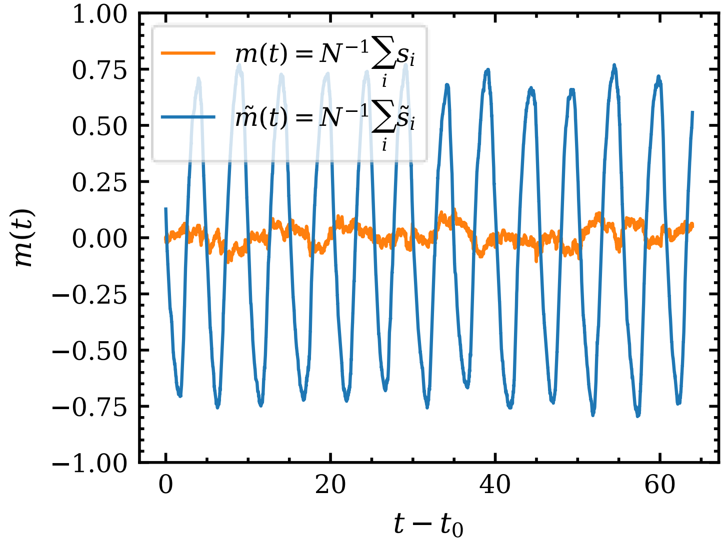
$$E_h = -\frac{1}{N}\left(J_2 \sum_{i,j} \tilde{h}_i \tilde{h}_j + (1-\mu) \sum_{i,j} \tilde{s}_i \tilde{h}_j\right)$$

We define new variables requiring the knowledge of the disorder

$\tilde{s}_i = \epsilon_i s_i$ and $\tilde{h}_i = \epsilon_i h_i$

$$\tilde{m} = N^{-1} \sum_i \epsilon_i s_i$$

The model in the variables $\tilde{s}_i$ and $\tilde{h}_i$ corresponds exactly to the mean-field model <u>without</u> disorder.



$T < T_c$ and $N = 470$

<u>Conclusion:</u>

With a perfect knowledge of the disorder, the oscillations are recovered.

Using this change of variable, we can explain analytically the previous results.

Conclusion

Starting from a out-of-equilibrium spin model exhibiting collective oscillations, we introduced quenched separable disorder and studied its effect numerically and analytically.

We showed that:

- The phase transition is hidden using the usual order parameters,
- The overlap distribution still carries the signature of an oscillating order,
- The oscillations can be recovered using a change of variable requiring full knowledge of the disorder.

Futur investigations: What happens with more complex disorder (non separable)? Does the hidden oscillating order remains?