Intermittent active dynamics at infinite persistence

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Disordered collective motion in passive (dead) matter

[Falk, Langer, 1998]

- In dense disordered packings, particles must move in a correlated way. In passive systems close to arrest, where crowding competes with thermal excitation, these rearrangements are the result of rare activated events.
- Their accumulation leads to disordered collective motion in the form of dynamical heterogeneity.

[Berthier, 2011]
Disordered collective motion in active (living) matter

Dense active systems (e.g., cell tissues, bacterial colonies), in which crowding competes with nonthermal driving, also display disordered collective motion, reminiscent of passive systems.

[Angelini et al., 2011]

- **Dense active systems** (e.g., cell tissues, bacterial colonies), in which crowding competes with nonthermal driving, also display disordered collective motion, reminiscent of passive systems.
Overdamped self-propelled particles: propulsion force \( p_i \) with correlation time \( \tau_p \) (\( \equiv \) persistence time).

\[
\begin{align*}
\xi \dot{r}_i &= p_i \\
\tau_p \dot{p}_i &= -p_i + \sqrt{2\xi^2 D_0} \eta_i
\end{align*}
\]
Overdamped self-propelled particles: propulsion force $p_i$ with correlation time $\tau_p$ (≡ persistence time).

Disordered mixture: pairwise interaction potential $U$ for polydisperse particles with packing fraction $\phi$ in 2D.

\[ \dot{\xi} \vec{r}_i = -\varepsilon \nabla_i U + p_i \]  
\[ \tau_p \dot{p}_i = -p_i + \sqrt{2\xi^2 D_0} \eta_i \]
Overdamped **self-propelled particles**: propulsion force $p_i$ with correlation time $\tau_p$ (≡ persistence time).

**Disordered mixture**: pairwise interaction potential $U$ for polydisperse particles with packing fraction $\phi$ in 2D.

Solve **overdamped Langevin dynamics** to study the competition between crowding and forcing.

\[
\dot{r}_i = -\nabla_i U + p_i \tag{1}
\]

\[
\tau_p \dot{p}_i = -p_i + \sqrt{2D_0} \eta_i \tag{2}
\]

**units**: length $\overline{\sigma} = 1$, time $\tau_0 = \xi \overline{\sigma}^2 / \varepsilon = 1$, energy $\varepsilon = 1$ \tag{3}
**Model**

- Overdamped self-propelled particles: propulsion force $p_i$ with correlation time $\tau_p$ (≡ persistence time).
- Disordered mixture: pairwise interaction potential $U$ for polydisperse particles with packing fraction $\phi$ in 2D.
- Solve overdamped Langevin dynamics to study the competition between crowding and forcing.

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\dot{\mathbf{r}}_i = -\nabla_i U + p_i \\
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\]

*units*: length $\bar{\sigma} = 1$, time $\tau_0 = \xi \bar{\sigma}^2 / \varepsilon = 1$, energy $\varepsilon = 1$

*control parameters*: $(D_0, \tau_p, \phi)$
Slow and persistent dynamics

\[ \dot{\mathbf{r}}_i = -\nabla_i U + \mathbf{p}_i \]  
\[ \tau_p \dot{\mathbf{p}}_i = -\mathbf{p}_i + \sqrt{2D_0} \eta_i \]  
\[ \Delta r^2(t) = \left< |\mathbf{r}_i(t) - \mathbf{r}_i(0)|^2 \right> \]

[YEK, Jack, Berthier, 2022]

- Both small ($\tau_p \ll \tau_0$) and large ($\tau_p \gg \tau_0$) persistence systems exhibit two-step relaxation scenario as packing fraction $\phi$ increases.
- Velocity variance drops ($\left< |\dot{\mathbf{r}}_i|^2 \right> \ll \left< |\mathbf{p}_i|^2 \right>$) indicating the system is close to force balance.
- Instantaneous kinetic energy shows hints of intermittent behaviour.
ADD scheme ($\tau_p \rightarrow \infty$)

$$t' = t/\tau_p \quad (8)$$

$$\frac{1}{\tau_p} \frac{dr_i}{dt'} = -\nabla_i \left( U - \sum_j r_j \cdot p_j \right) \quad (9)$$

$$\frac{dp_i}{dt'} = -p_i + \sqrt{2D_0/\tau_p \eta_i'} \quad (10)$$
ADD scheme ($\tau_p \to \infty$)

\[ t' = t/\tau_p \]  \hspace{1cm} (8)

\[ 0 = -\nabla_i U_{\text{eff}}[r_j, p_j] \]  \hspace{1cm} (9)

\[ \frac{dp_i}{dt'} = -p_i + \sqrt{2f} \eta'_i \]  \hspace{1cm} (10)

\[ \delta \epsilon_p \geq 0 \]

\[ \delta \epsilon_p < 0 \]

[YEK, Mandal, et al., 2022]
ADD scheme ($\tau_p \to \infty$)

\[ t' = t / \tau_p \]

\[ 0 = -\nabla_i U_{\text{eff}}[r_j, p_j] \]

\[ \frac{dp_i}{dt'} = -p_i + \sqrt{2f} \eta'_i \]

- $U_{\text{eff}}(0)$
- $U_{\text{eff}}(\delta t')$

\[ \delta \epsilon_p \geq 0 \quad \delta \epsilon_p < 0 \]

[YEK, Mandal, et al., 2022]

- Intermittent dynamics similar to AQS [Maloney, Lemaître, 2006] and AQRD [Morse et al., 2021] without deformation of the system.

- Propulsions forces are updated \textit{quasistatically} similary to AQRF [Morse et al., 2021], however this update is \textit{stochastic} and the system supports an isotropic dynamical steady state.
Average dynamics

\[ 0 = -\nabla_i U_{\text{eff}}[\mathbf{r}_j, \mathbf{p}_i] \]
\[ \frac{d\mathbf{p}_i}{dt'} = -\mathbf{p}_i + \sqrt{2f}\eta'_i \]
\[ \Delta r^2(t) = \langle |\mathbf{r}_i(t) - \mathbf{r}_i(0)|^2 \rangle \]

- Dynamics is **diffusive** at small and large times.
- Dynamics speeds up with increasing \( N \): there are more frequent and larger plastic events.
0 = −\nabla_i U_{\text{eff}} [\mathbf{r}_j, \mathbf{p}_i] \quad (14)

\chi_b(t') = \text{Var}(\text{fraction of initially close particles still close at } t') \quad (15)

- Relaxation is slower and more heterogeneous as \( f \) is decreased.

- Structural relaxation events tend to happen in the vicinity of previous ones: dynamical facilitation?
Thank you for listening!

\[ \sum_{t'' \leq t'} \delta \epsilon_p(t'') \leq t' \delta \nu \]

\[ f = 0.9 \]

\[ C_{b,i}(t' = 3.86) \]

\[ C_{b,i}(t' = 5.02) \]

\[ C_{b,i}(t' = 6.53) \]

\[ C_{b,i}(t' = 8.49) \]

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Intermittent active dynamics at \( \tau_p \rightarrow \infty \)

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Maloney, Craig E., Anaël Lemaître (July 2006). “Amorphous Systems in Athermal, Quasistatic Shear”. In: *Physical Review E* 74.1. ISSN: 1539-3755, 1550-2376.


Elastic displacement correlations

![Graph showing correlations with different values of N and f]

- $g_{ADD}$
- $g_{AQS}$

Parameters:
- $N = 500$
- $N = 1024$
- $N = 2000$
- $f = 0.9$
Plastic events distributions

\[ \text{Prob}(\log_{10} \tau' N^{1.5}) \]

\[ \langle \tau' \rangle \sim N^{-1.5} \]

\[ \text{Prob}(\log_{10} S) \sim N^{0.675} \]

\[ f = 0.9 \]

\[ N = 500 \]
\[ N = 1024 \]
\[ N = 2000 \]

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Intermittent active dynamics at \( \tau_p \to \infty \)
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Displacement distribution

\[ \tilde{G}_s(r) \sim e^{-\Delta r_{x,y}^2/\text{MSD}} \]

\[ f = 0.9 \]
Proportion of unbroken bonds

\[ C_b(t') \sim e^{-\left(\frac{t}{\tau'}\right)^{0.75}} \]

\[ f = 0.8 \]
\[ f = 0.825 \]
\[ f = 0.85 \]
\[ f = 0.875 \]
\[ f = 0.9 \]

\[ \tau_p \to \infty \]

\[ N = 500 \]
\[ N = 1024 \]
\[ N = 2000 \]

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