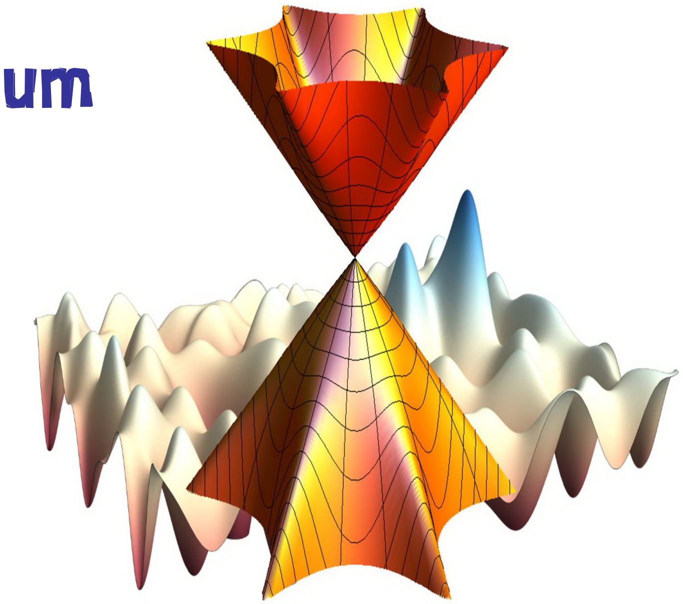
Interaction, disorder, elasticity GDR meeting (Grenoble)
28 November 2022

Non-Anderson disorder-driven quantum transition in nodal semimetals



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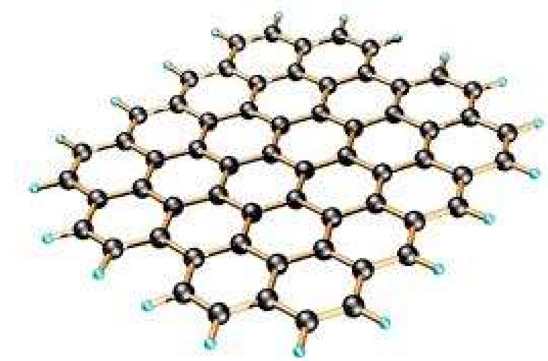
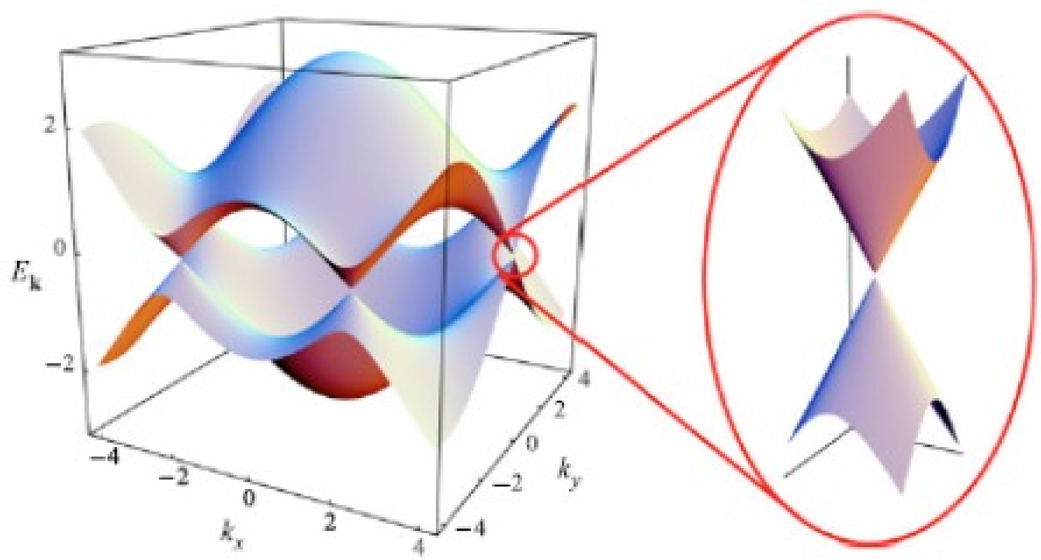
Nodal semimetals: linear energy band crossing

Seminal example: Graphene (2d hexagonal lattice of Carbon atoms, 1 p_z orbital per site)

K. S. Novoselov, et al, Nature 438 , 197 (2005)





Emergent Dirac Fermions

Low energy Hamiltonian

$$\hat{H} = -i\hbar v_F(\sigma_x \partial_x + \sigma_y \partial_y)$$

mass

$$m = 0$$

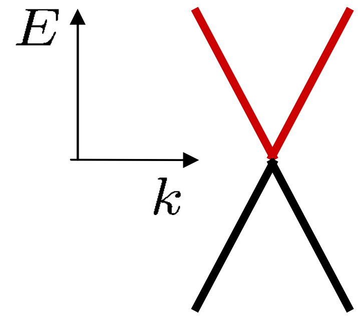
Fermi velocity

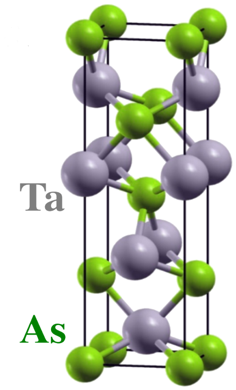
$$v_F \approx 8 \times 10^5 m/c$$

Beyond Graphene: 3D nodal semimetals

2 bands crossing in D=3: Weyl semimetal
discovered in TaAs, TaP, NbAs, NbP,...

$$\hat{H}_{\mathrm{Weyl}} = -i\hbar v_F(\sigma_x\partial_x + \sigma_y\partial_y + \sigma_z\partial_z)$$

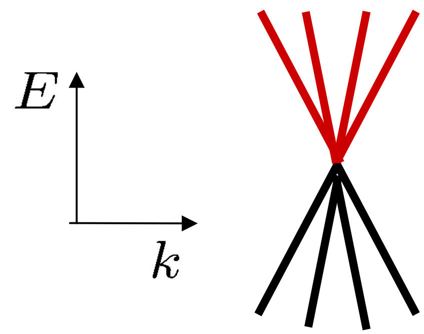


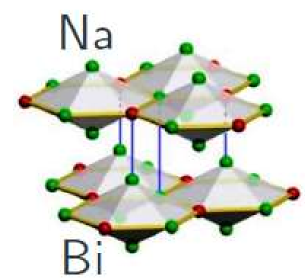


S.-Y. Xu et al., “Discovery of a Weyl Fermion Semimetal and Topological Fermi Arcs,” Science (2015)
B. Q. Lv et al., “Experimental Discovery of Weyl Semimetal TaAs,” Phys. Rev. X (2015)
L. Lu, et al., “Experimental Observation of Weyl Points,” Science (2015).

4 bands crossing in D=3: Dirac semimetal
discovered in Na3Bi, Cd3As2, ...

Wang et al. (2012), Liu et al. (2014), Xu et al. (2015)
Wang et al. (2013), Neupane et al. (2014), Borisenko et al. (2014)





$$\hat{H}_{\mathrm{Dirac}} = \begin{pmatrix} H_{\mathrm{Weyl}} & 0 \\ 0 & -H_{\mathrm{Weyl}} \end{pmatrix}$$ **Weyl representation**

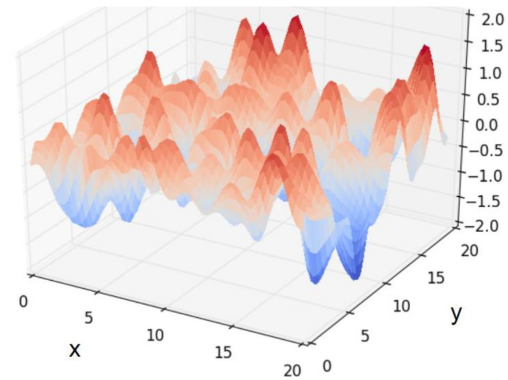
Anderson localization tranistion

Single electron in a Gaussian random potential

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{x})\right]\psi_\alpha(\boldsymbol{x}) = E_\alpha\psi_\alpha(\boldsymbol{x})$$

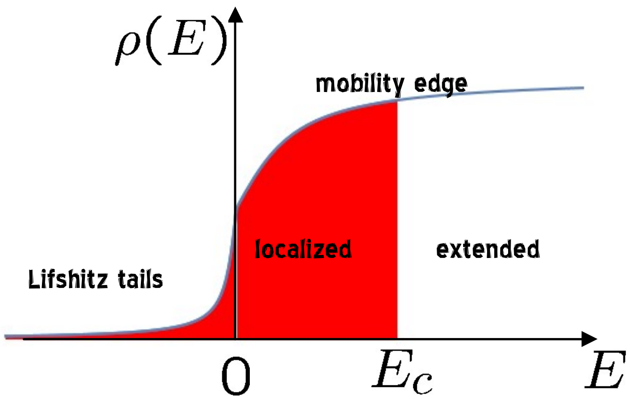
$$\overline{V(\boldsymbol{x})} = 0$$

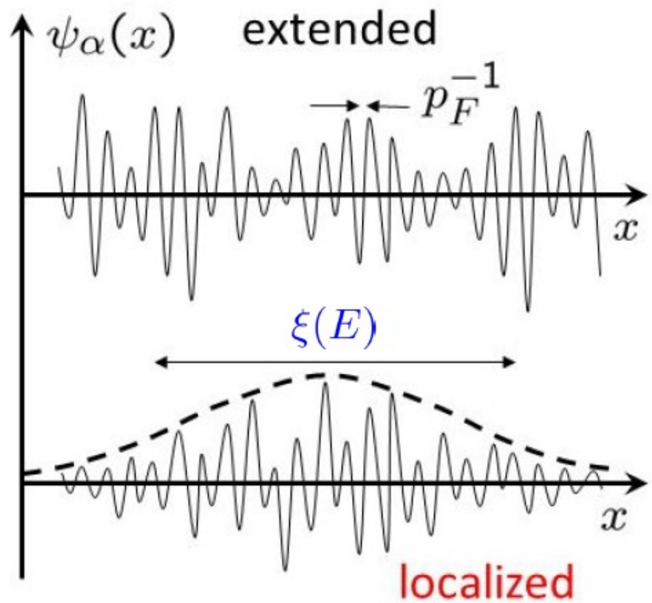
$$\overline{V(\boldsymbol{x})V(\boldsymbol{x}')} = \Delta\delta(\boldsymbol{x}-\boldsymbol{x}')$$



Density of states

$$\rho(E) = \frac{1}{\text{volume}}\sum_\alpha (E - E_\alpha)$$





Localization length

$$\xi \sim (E_c - E)^{-\nu}$$

Conductivity

$$\sigma \sim (E - E_c)^s \quad s = \nu(d-2)$$

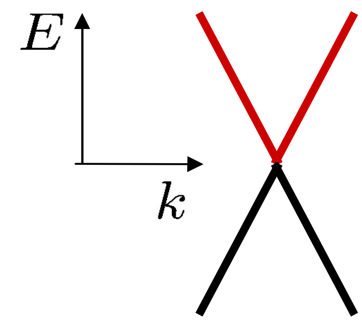
$\nu = 1.57 \pm 0.02$ **(3D Orthogonal)**

K. Slevin, T. Ohtsuki, New J. Phys. 16, 015012 (2014)

Effect of disorder on relativistic electrons (heuristic picture)

Single Weyl cone Hamiltonian

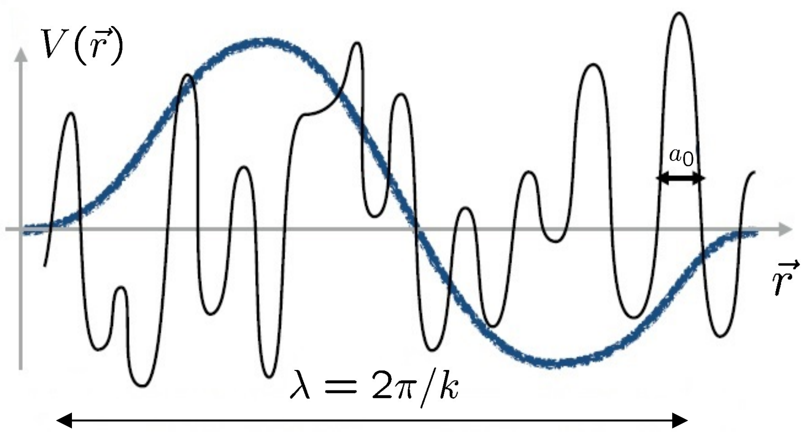
$$\hat{H} = -i\hbar v_F \sigma_j \partial_j + V(x)$$



Gaussain random potential :

$$\overline{V(x)} = 0 \qquad \overline{V(x)V(x')} = \Delta\delta(x - x')$$

Scaling arguments

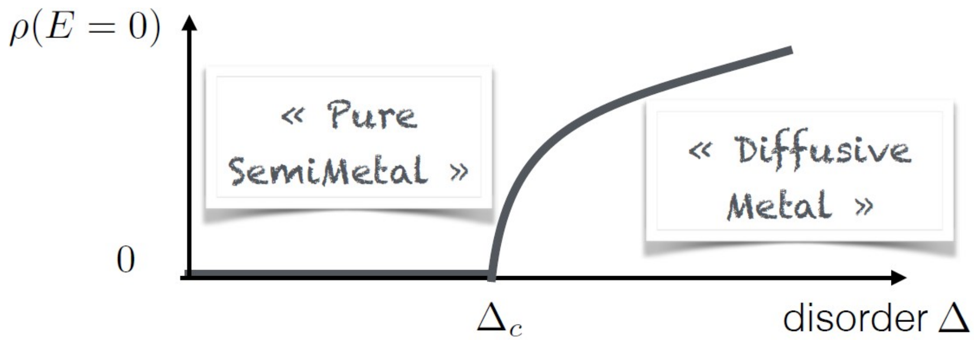


Kinetic energy : $E_{typ} = \hbar v_F k$

Disorder potential : $V_{typ} \sim \sqrt{\Delta}\left(\frac{\lambda}{a_0}\right)^{-d/2}$

In the limit of zero energy ($k \to 0$)
disorder is dominant for $d < 2$
and irrelevant for $d > 2$

New disorder driven quantum transition $\neq$ Anderson localization



$$\rho(0,\Delta) \sim |\Delta - \Delta_c|^{\beta}$$

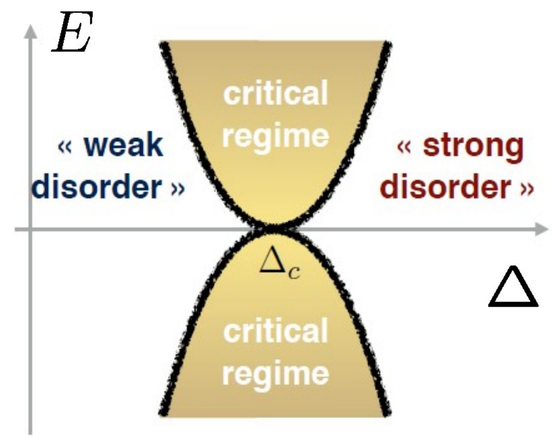
$$\beta = \nu(d-z)$$

The mean free path:

$$\xi(E,\Delta) = |\Delta - \Delta_c|^{-\nu}\tilde{\xi}(E|\Delta - \Delta_c|^{-\nu z})$$

Density of states (DOS) :

$$\rho(E,\Delta) = \xi^{z-d}\tilde{\rho}(E\xi^{z}, |\Delta - \Delta_c|\xi^{1/\nu})$$



K. Kobayashi, et al., PRL 112, 016402 (2014)

What is the theory of the transtion ?

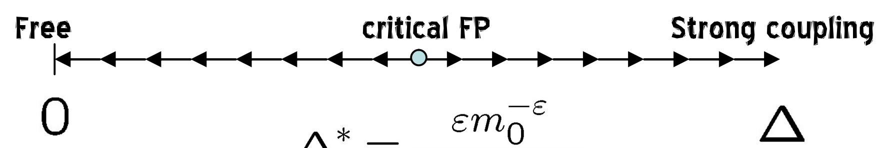
Action at zero energy: $S = i\int d^dx\,\bar{\psi}(x)(-i\sigma_j\partial_j + V(x))\psi(x) \quad (\hbar v_F = 1)$

Replica trick: **Gross-Neveu model**

$$S_{\mathrm{GN}} = \int d^dx \left[\sum_{\alpha=1}^{N} \bar{\psi}_\alpha \sigma_j \partial_j \psi_\alpha + \frac{1}{2}\Delta \sum_{\alpha,\beta=1}^{N} \left(\bar{\psi}_\alpha\psi_\alpha\right)\left(\bar{\psi}_\beta\psi_\beta\right)\right]$$

Renormalization group flow in $d = 2+\varepsilon,\ \Delta = \tilde{\Delta} m^{-\varepsilon}$

$$-m\partial_m\tilde{\Delta} = -\varepsilon\tilde{\Delta} + 4(1-N)\tilde{\Delta}^2 + \ldots$$



$$\Delta^* = \frac{\varepsilon m_0^{-\varepsilon}}{4(1-N)}$$

Generation of the $\bar{\psi}_\alpha\psi_\alpha$ **term**

$N > 1,\ \Delta^* < 0$

d **is space-time**

Chiral transition

(fermionic mass generation)

V.S.

$N \to 0,\ \Delta^* > 0$

d **is only space**

Semimetal - metal transition

(generation of DOS at zero energy)

Problems with the Gross-Neveu description

$d = 2 + \varepsilon$ expansion

1-loop	$\nu = 1$	P. Goswami S. Chakravarty, PRL 107, 196803 (2011)
2-loop	$\nu = 0.67$	B. Roy and S. Das Sarma, PRB 90, 241112(R) (2014) S. V. Syzranov, P. M. Ostrovsky, V. Gurarie, L. Radzihovsky, PRB 93, 155113 (2016)
3-loop	$\nu = 0.53$	T. Louvet, AAF, D. Carpentier, PRB 94, 220201(R) (2016)
4-loop	$\nu = 0.108$	J.A. Gracey, T. Luthe, Y. Schroder, PRD 94, 125028 (2016)

Numerics in 3D

$\nu = 0.99 \pm 0.05$	B. Roy, R.J. Slager, V. Juričić PRX 8, 031076 (2018)
$\nu = 1.47 \pm 0.03$	B. Sbierski, E. J. Bergholtz, P. W. Brouwer, PRB 92, 115145 (2015)
$\nu = 0.92 \pm 0.13$	K. Kobayashi, T. Ohtsuki, K.-I. Imura, I. F. Herbut, PRL 112, 016402 (2014)

Functional renormalization group (FRG)

I. Balog, D. Carpentier, AAF, PRL 121, 166402 (2018)

Averaging over arbitrary distribution of spatially uncorrelated disorder

$$\overline{\exp(-i\int d^dx V(x)\Theta(x))} = \exp(-\int d^dx W(\Theta(x)))$$

$$W(\Theta) = \frac{1}{2}\Delta\Theta^2 + \text{higher cumulants}$$ **is the characteristic function**

Replicated action

$$S = \int d^dx \sum_{\alpha=1}^{N} \bar{\psi}_\alpha(x)\sigma_j\partial_j\psi_\alpha(x) + W(\Theta(x))$$

$$\Theta(x) = \sum_{\alpha=1}^{N} \bar{\psi}_\alpha(x)\psi_\alpha(x)$$ **is the local density of fermions**

FRG flow equation (the IR cutoff flows from $m = m_0$ to $m = 0$):

$$-m\partial_m W(\Theta) = 2m^\varepsilon \left(\Theta W'(\Theta)W''(\Theta) - NW'(\Theta)^2\right)$$

Analysis of FRG

FRG flow equation (the IR cutoff flows from $m = m_0$ to $m = 0$):

$$-m\partial_m W(\Theta) = 2m^{\varepsilon} \left(\Theta W'(\Theta)W''(\Theta) - NW'(\Theta)^2\right)$$

We look for a fixed point solution of the form $W(\Theta) = m^{2+\varepsilon} w(\Theta m^{-1-\varepsilon})$

$$-m\partial_m w(\theta) = (2+\varepsilon)w(\theta) - (1+\varepsilon)\theta w'(\theta) + 2\left(\theta w'(\theta)w''(\theta) - Nw'(\theta)^2\right)$$

The Gross-Neveu fixed point

$$w^*(\theta) = \varepsilon\theta^2/[8(1-\tilde{N})]$$

Linearization around the GN FP gives the eigenvalues:

$$[w^{(n)}(0)] = 2+\varepsilon-n(1+\varepsilon)+\varepsilon n(2N-n)/(2N-2)$$

Singly unstable in the limit $N \to \infty$ and infinitely unstable in the limit $N \to 0$!

FRG flow equation in large N limit **(the IR cutoff flows from** $m = m_0$ **to** $m = 0$ **)**

$$-m\partial_m W(\Theta) = -2m^{\varepsilon} N W'(\Theta)^2$$

„time“: $t = \dfrac{(m_0^{\varepsilon} - m^{\varepsilon})}{\varepsilon} \leq T_0 = \dfrac{m_0^{\varepsilon}}{\varepsilon}$

„coordinate“: $\Theta = 4Nr$

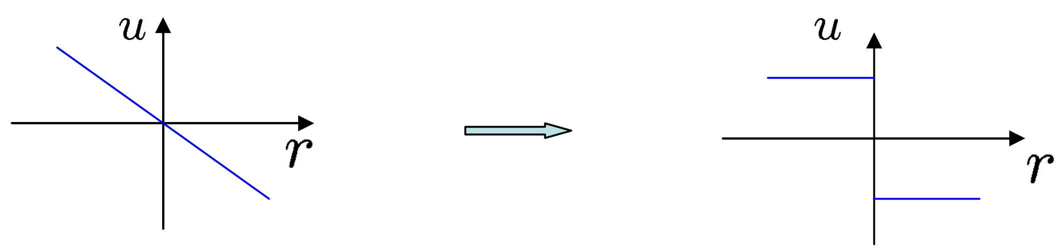
„velocity“: $u = W'(\Theta)$

Burgers equation

$$\partial_t u(r) + u(r)u'(r) = 0$$

After waiting (breaking) time $T^* = 1/|u_0'(0)|$ **and if** $T^* \leq T_0$ **(max. time)** **the "velocity" profile** $u(r)$ **develops a shock exactly at the origin** $r = 0$

Critical point $T^* = T_0 \longrightarrow \frac{m_0^{\varepsilon}}{\varepsilon} = \frac{1}{4N|\Delta|} \longrightarrow |\Delta^*| = \frac{\varepsilon m_0^{-\varepsilon}}{4N}$



The shock leads to $W'(0^+) \neq 0$ **and thus, to the dynamical fermion mass generation (term** $\bar{\psi}\psi$ **in the action!)**

Disordered Weyl fermions in the limit of $N \to 0$

FRG flow equation (the IR cutoff flows from $m = m_0$ to $m = 0$):

$$-m\partial_m W(\Theta) = 2m^{\varepsilon}\Theta W'(\Theta)W''(\Theta)$$

„time“: $t = \frac{(m_0^{\varepsilon} - m^{\varepsilon})}{\varepsilon} \leq T_0 = \frac{m_0^{\varepsilon}}{\varepsilon}$

„coordinate“: $r = \sqrt{2\Theta}$

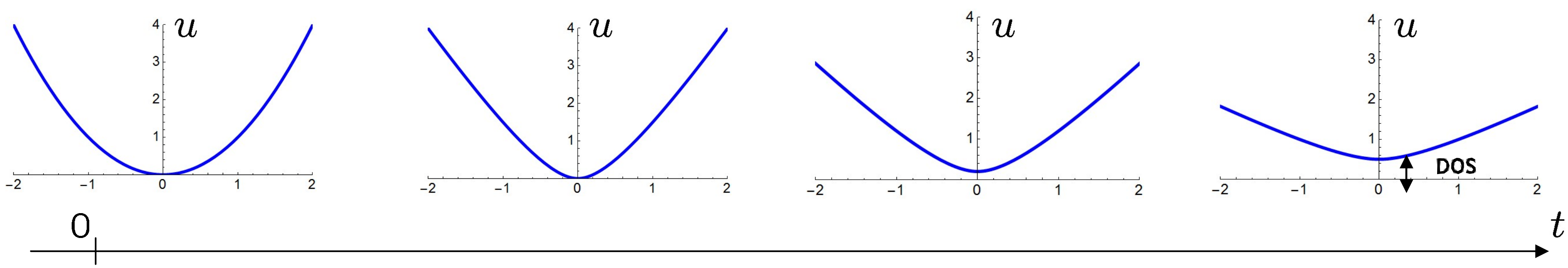
„density“: $u = W'(\Theta)$

2D Porous Medium Equation (PME)

$$2\partial_t u(r,t) = \frac{1}{r}\partial_r r \partial_r u^2(r,t) = \Delta u^2(r,t)$$

J. L. Vazquez, The Porous Medium Equation (Clarendon Press, Oxford, 2007)

Transition is a waiting time phenomenon in non-linear diffusion



Waiting time before the density starts to accumulate at $r = 0$

$$T_c \sim 1/\text{disorder strength}$$

Critical point $T_c = T_0$

Theory of Porous medium equation

$$2\partial_t u(r,t) = \frac{1}{r}\partial_r r \partial_r u^2(r,t) = \Delta u^2(r,t)$$

The large time behavior is given by the self-similar solutions

$$\delta = (1+\varepsilon)/(2\varepsilon)$$

Backward self-similar solution

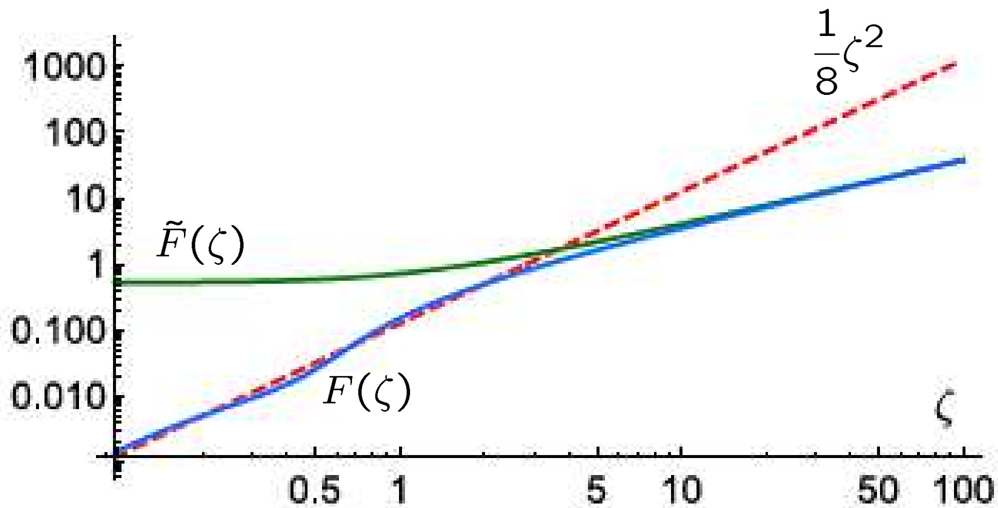
$$u(r,t) = (T_c - t)^{2\delta-1} F\left(\frac{r}{(T_c-t)^\delta}\right)$$

$$t < T_c$$

Forward self-similar solution

$$u(r,t) = (t - T_c)^{2\delta-1} \tilde{F}\left(\frac{r}{(t-T_c)^\delta}\right)$$

$$t > T_c$$



$$\delta = \varepsilon = 1$$

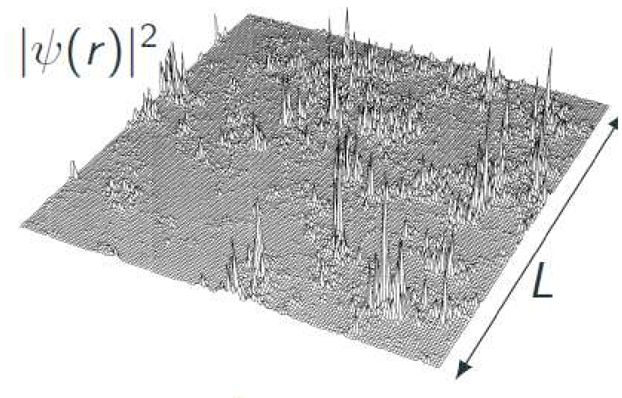
$$u(r, t = T_c^-) = u(r, t = T_c^+)$$

One loop exponents

$$\nu = \frac{1}{\varepsilon} \qquad \beta = 2\delta - 1$$

Multifractality at the nodal semimetal – diffusive metal transition

Critical wave functions display multifractal behaviour



Inverse participation ratio $P_q = \int \mathrm{d}^d r \overline{|\psi(\boldsymbol{r})|^{2q}}$

$$P_q \sim \begin{cases} L^{-d(q-1)} & \text{(extended states)} \\ L^{-d(q-1)-\tilde{\Delta}_q} & \text{(critical wave functions)} \\ L^0 & \text{(localized states)} \end{cases}$$

Multifractal spectrum

$$\tilde{\Delta}_q^{\mathrm{Weyl}} = \frac{3}{8} q(1-q)\varepsilon^2 + \mathcal{O}(\varepsilon^3)$$

T. Louvet, AAF, D. Carpentier, PRB 94, 220201(R) (2016)

Anderson localization

$$\tilde{\Delta}_q^{(\mathrm{O})} = q(1-q)\varepsilon + \mathcal{O}(\varepsilon^4)$$

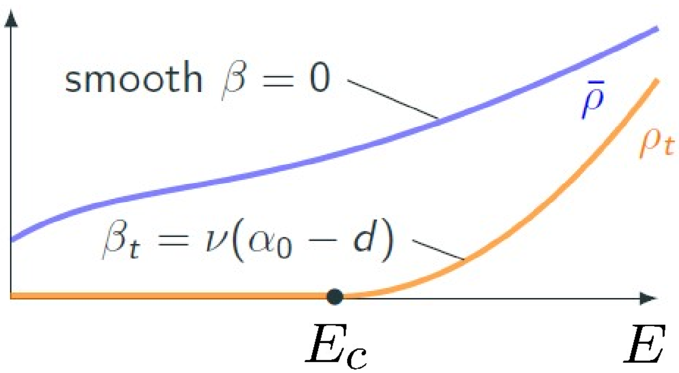
$$\tilde{\Delta}_q^{(\mathrm{U})} = q(1-q)\sqrt{\varepsilon/2} + \mathcal{O}(\varepsilon^2)$$

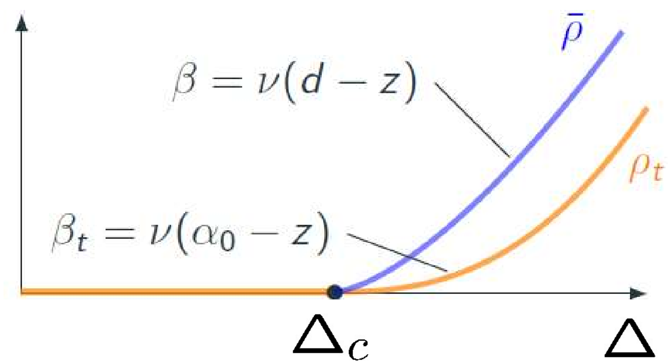
The multifractal spectrum is related to the scaling dimension of the composite operator

$$\mathcal{O}_q(\boldsymbol{r}) = \prod_{\alpha=1}^{q} |\psi_\alpha(\boldsymbol{r})|^2 \sim L^{-x_q} \qquad \tilde{\Delta}_q = x_q - q x_1$$

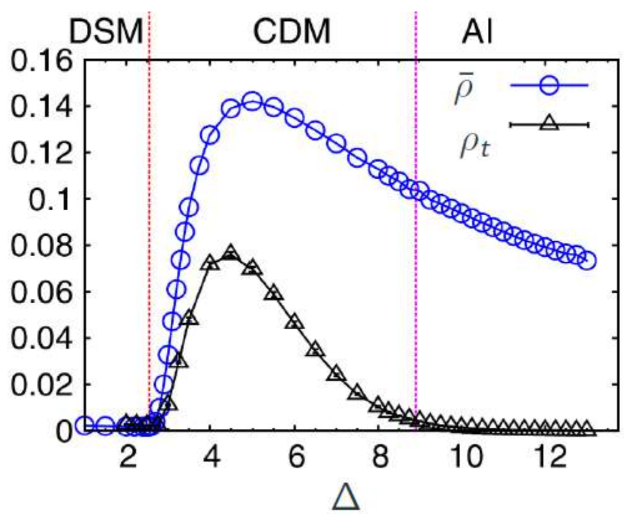
Typical DOS $\rho_t = \exp \overline{\ln \rho(\mathbf{r})}$ vs averaged DOS $\overline{\rho(\mathbf{r})}$

Anderson localization | **Semimetal-metal transition**





Legendre transform of the multifractal spectrum = singularity spectrum $\tilde{f}(\alpha)$ which has maximum at $\alpha = \alpha_0$



J. H. Pixley, P. Goswami, S. Das Sarma, PRL 115, 076601 (2015)

$$\left.\begin{aligned} \beta &= 1.4 \pm 0.2 \\ \beta_t &= 2.0 \pm 0.3 \\ z &= 1.46 \pm 0.05 \end{aligned}\right\} \alpha_0 = 3.7 \pm 0.6$$

Two-loop [1/1] Pade $\alpha_0 = 3.6$

E. Brillaux, D. Carpentier, AAF, PRB 100, 134204 (2019)

Surface criticality of disordered Dirac fermions

E. Brillaux, D. Carpentier, AAF, PRB 100, 103, 081405 (2021)

Semi-infinite system

$$\hat{H}_0 = i\tau_z \boldsymbol{\sigma}\cdot\boldsymbol{\partial} \qquad z > 0$$

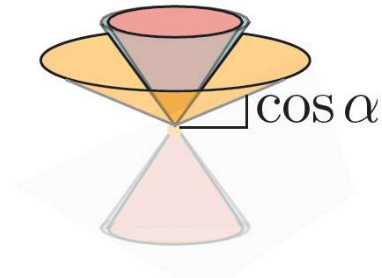
Boundary conditions $\quad M\psi|_{z=0^+} = \psi|_{z=0^+}$

$$M = \begin{pmatrix} 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \\ e^{-i\alpha} & 0 & 0 & 0 \\ 0 & -e^{i\alpha} & 0 & 0 \end{pmatrix}$$

Surface states

$$\hat{H}_0\psi = \epsilon\psi \qquad \psi \sim e^{-\mu z}$$

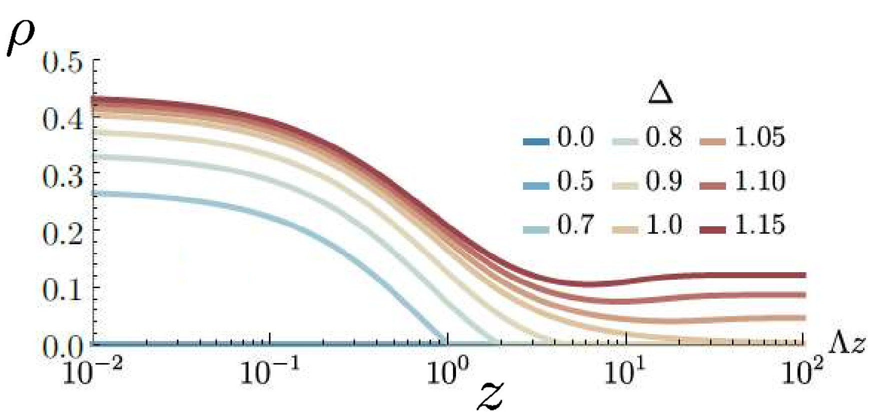
$$\varepsilon = k_{\parallel} \cos\alpha$$
$$\mu = k_{\parallel} \sin\alpha$$

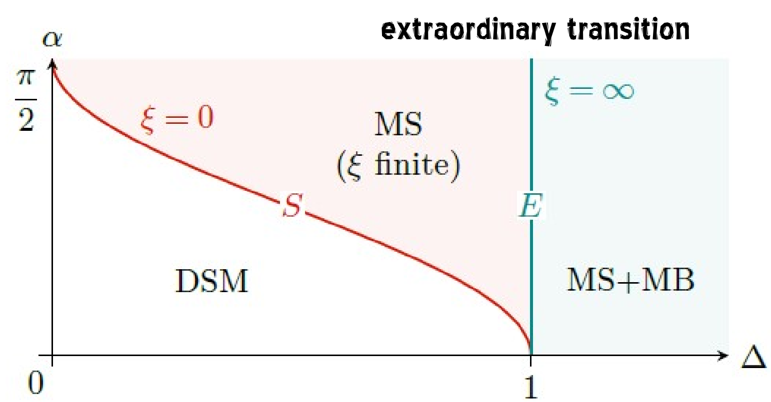


Phase diagram in the presence of bulk disorder

$$\hat{H} = i\tau_z \boldsymbol{\sigma}\cdot\boldsymbol{\partial} + V(\boldsymbol{x})$$
$$\overline{V(\boldsymbol{x})V(\boldsymbol{x}')} = \Delta\delta(\boldsymbol{x}-\boldsymbol{x}')$$

Self-consistent Born approximation





Conclusions

- Nodal semimetal- diffusive metal transiton is a non-Anderson quantum transition driven by disorder

- It exhibits many properties of the Anderson transition but their manifistation is weaker while its description is simpler

- FRG provides acess not only to the critical point but also to the strong coupling phase